MATHEMATICS

Sixth Form Entry September 2023

Time: $1\frac{1}{2}$ hours

Name: ....…..……

School: ....….

Instructions
- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- **Calculators are allowed in this examination.**
- Answer the questions in the spaces provided – *there may be more space than you need.* Ensure that your answers to parts of questions are clearly labelled.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information

- There are 25 questions in this examination and the total mark is 100.
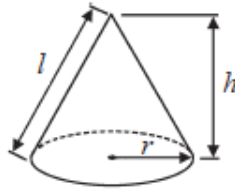- Check your answers if you have time at the end.
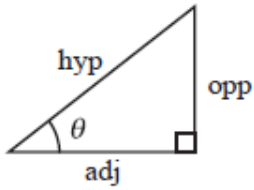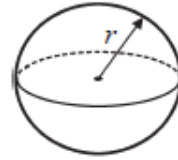
FORMULA SHEET

Pythagoras' Theorem
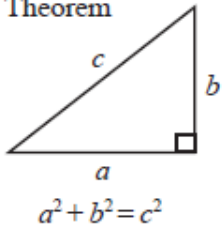
$a^2 + b^2 = c^2$

Volume of cone $= \frac{1}{3}\pi r^2 h$

Curved surface area of cone $= \pi r l$

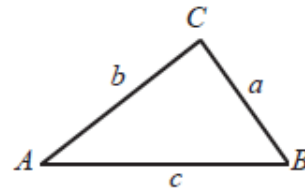Volume of sphere $= \frac{4}{3}\pi r^3$

Surface area of sphere $= 4\pi r^2$

$\text{adj} = \text{hyp} \times \cos\theta$
$\text{opp} = \text{hyp} \times \sin\theta$
$\text{opp} = \text{adj} \times \tan\theta$

or

$\sin\theta = \dfrac{\text{opp}}{\text{hyp}}$

$\cos\theta = \dfrac{\text{adj}}{\text{hyp}}$

$\tan\theta = \dfrac{\text{opp}}{\text{adj}}$

In any triangle ABC

Sine rule: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

Cosine rule: $a^2 = b^2 + c^2 - 2bc\cos A$

Area of triangle $= \frac{1}{2}ab\sin C$
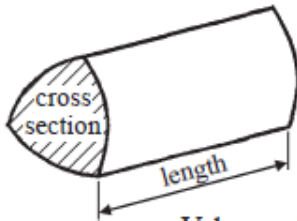
Volume of prism = area of cross section × length

Circumference of circle $= 2\pi r$

Area of circle $= \pi r^2$

Area of a trapezium $= \frac{1}{2}(a+b)h$
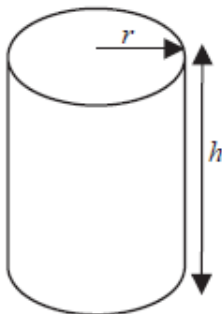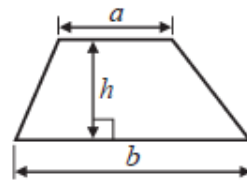
Volume of cylinder $= \pi r^2 h$

Curved surface area of cylinder $= 2\pi rh$

The Quadratic Equation
The solutions of $ax^2 + bx + c = 0$, where $a \neq 0$, are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Q1.

(a) Factorise $3y^2 + 2y$

.... (1)

(b) Expand and simplify $(x - 9)(x + 2)$

.... (2)

(Total for question = 3 marks)

_____

Q2. Here are the first five terms of an arithmetic sequence.

$$7 \qquad 10 \qquad 13 \qquad 16 \qquad 19$$

Find an expression for the nth term of the sequence.

....

(Total for question = 2 marks)

_____

Q3.

Show that $4\frac{1}{9} \div \frac{5}{18} = 14\frac{4}{5}$

(Total for question = 3 marks)

_____

Q4.

Make q the subject of the formula $p = \sqrt{q} - 5r$

$q = $ .... (2)

(Total for Question is 2 marks)

Q5.

(a) Write down the value of a^0

.... (1)

Given that $\dfrac{y^5 \times y^n}{y^6} = y^{13}$

(b) work out the value of *n*.

$n = $ .... (2)

(c) Simplify $(16y^8)^{\frac{3}{4}}$

.... (2)

(Total for question = 5 marks)

Q6.

(a)   On the grid, draw the line with equation $x + 2y = 8$ for values of x from 0 to 9
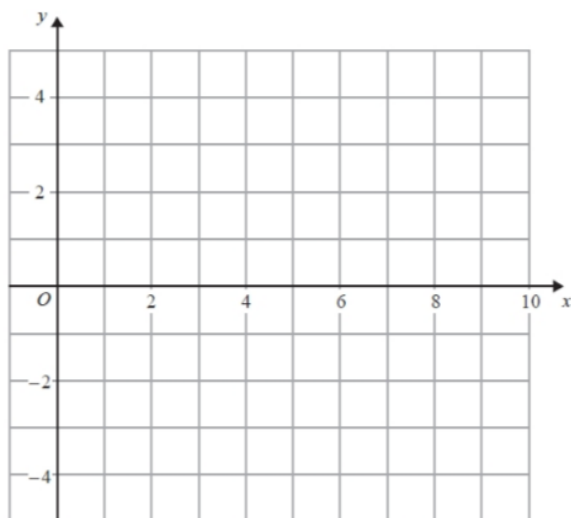
(2)

(b)   Show, by shading on the grid, the region defined by all three inequalities

$x + 2y \leq 8$
$x \geq 2$
$y \geq 1$

Label your region **R**.

(3)

(Total for Question is 5 marks)

Q7.

(a) Factorise $x^2 + 5x - 36$

....

(2)

(b) **Hence**, solve $x^2 + 5x - 36 = 0$

....

(1)

(Total for question = 3 marks)

Q8.
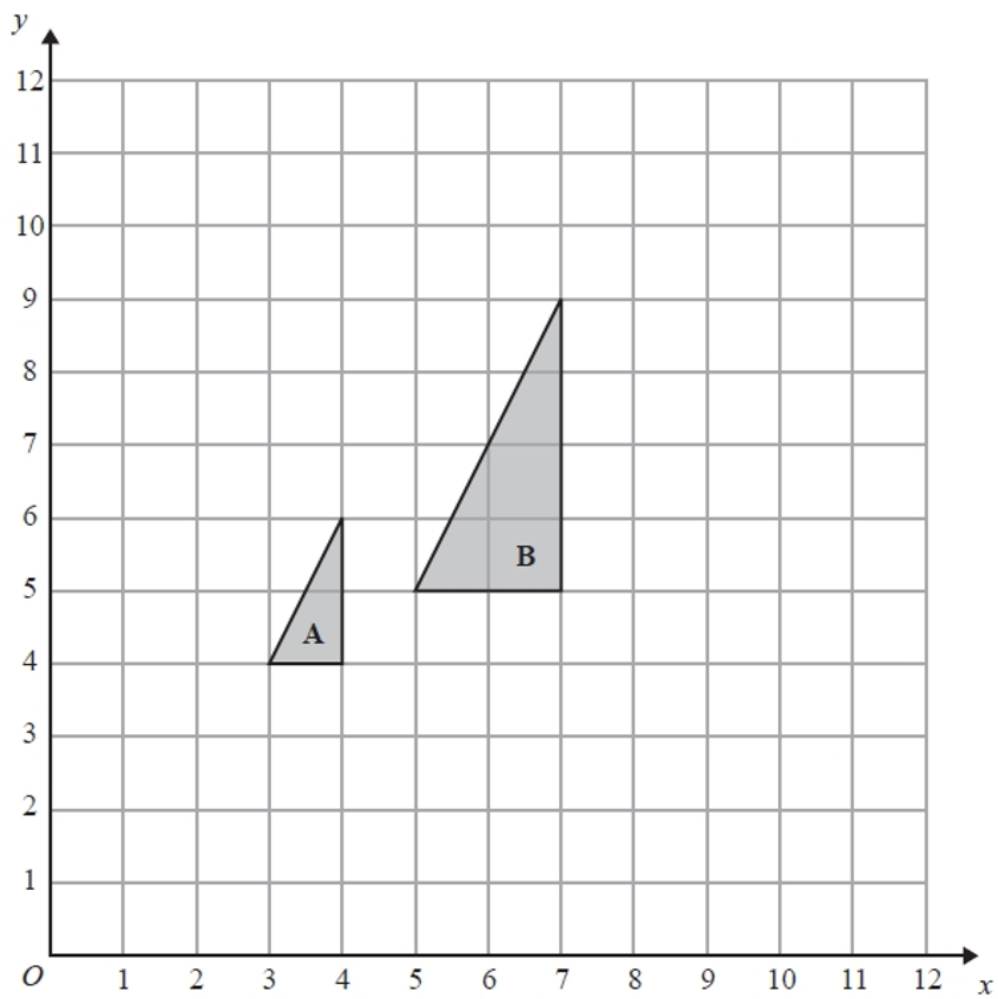
(a) Describe fully the single transformation that maps triangle **A** onto triangle **B**.

....

....

(3)

(b) On the grid, translate triangle **A** by the vector $\begin{pmatrix} 5 \\ -2 \end{pmatrix}$

(1)

(Total for question = 4 marks)

Q9.

(a) Complete the table of values for $y = x^2 - 3x - 1$

x	-2	-1	0	1	2	3	4
y			-1		-3		3

(2)

(b) On the grid, draw the graph of $y = x^2 - 3x - 1$ for all values of x from -2 to 4
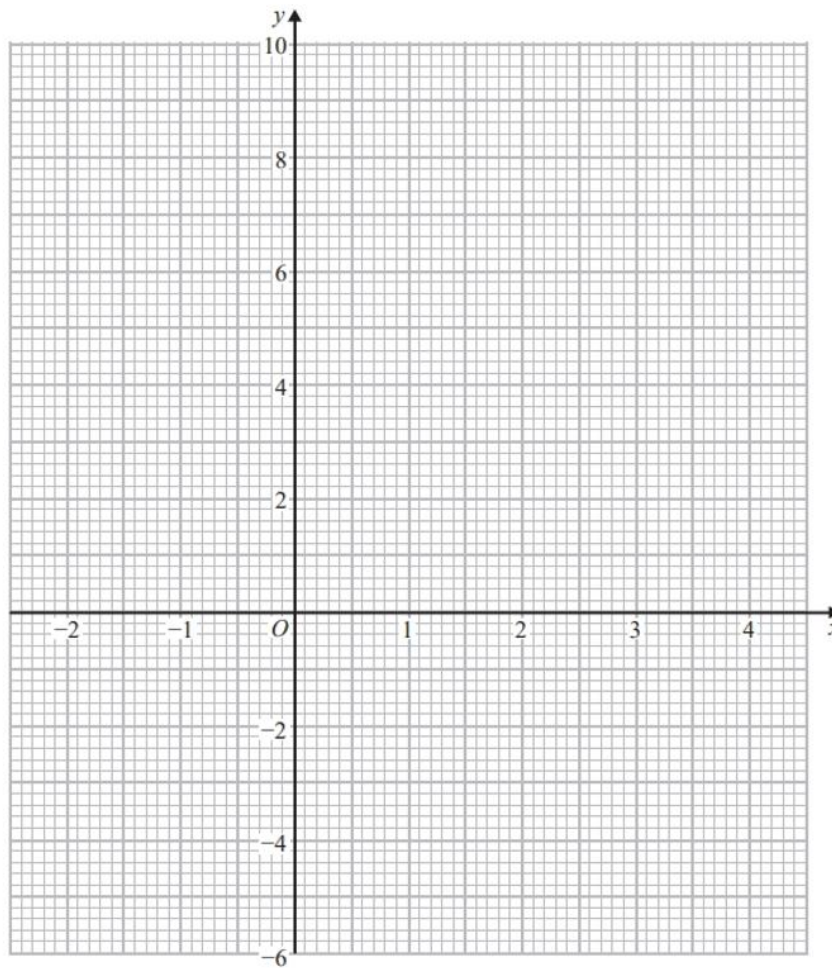
(2)

(c) Use the graph of $y = x^2 - 3x - 1$ to solve the equation $x^2 - 3x - 1 = 0$

.... (1)

(Total for question = 5 marks)

Q10.

In a game, a fair dice is rolled once and a fair spinner is spun once.

The dice has dots showing 1, 2, 3, 4, 5 and 6
The spinner has three sections numbered 1, 2 and 3

The score is found by subtracting the smaller number from the larger number.

(a) Complete the sample space diagram to show all the possible scores.

Seven scores are shown for you.

		Dice					
		1	**2**	**3**	**4**	**5**	**6**
	1	0	1	2			
Spinner	**2**	1	0				
	3			0		2	

(2)

Pau plays the game.

(b) Find the probability that his score is

(i) 5

....

(ii) 2 or 4

....
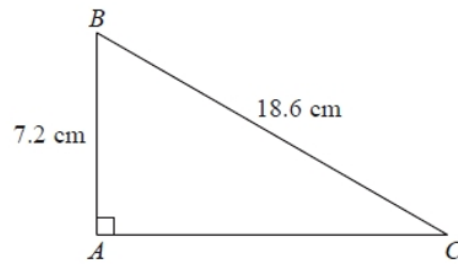
(2)

(Total for question = 4 marks)

Q11.

(a)

B

7.2 cm

18.6 cm

A

C

Diagram **NOT**
accurately drawn

Calculate the length of *AC*.
Give your answer correct to 3 significant figures.

.... cm (3)

(b)

7.8 cm

x cm

43°
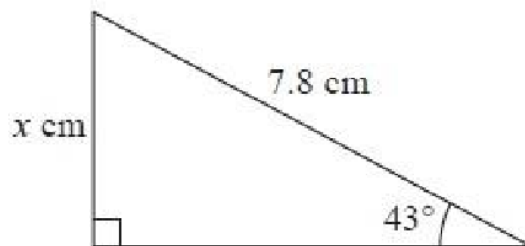
Diagram **NOT**
accurately drawn

Work out the value of x.

Give your answer correct to 3 significant figures.

x = ... (3)

(Total for Question is 6 marks)

Q12.

Expand and simplify $(2x - 1)(x + 3)(x - 5)$

....

(3)

(Total for question = 3 marks)

_____

Q13.

Diagram **NOT** accurately drawn
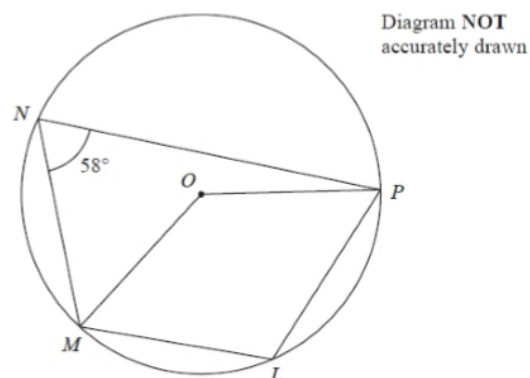
L, M, N and P are points on a circle, centre O. Angle $MNP = 58°$

(a) (i) Find the size of angle MLP

....°

(ii) Give a reason for your answer.

.... (2)

(b) Find the size of the reflex angle MOP

....° (2)

(Total for question = 4 marks)

Q14.

The diagram shows the positions of two towns, *A* and *B*.

North

B

Diagram **NOT**
accurately drawn

110 km

North

x

A   60 km
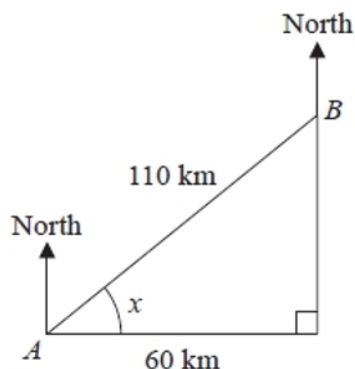
The distance from *A* to *B* is 110 km.
B is 60 km east of *A*.

(a) Work out the size of angle *x*.

Give your answer correct to 1 decimal place.

.... °

(3)

(b) Work out the bearing of *B* from *A*.

Give your answer correct to the nearest degree.

.... °

(2)

The distance from *A* to *B* is 110 km correct to **2 significant figures**.

(c) (i) Write down the lower bound for the distance from *A* to *B*.

.... km

(ii) Write down the upper bound for the distance from *A* to *B*.

.... km

(2)

(Total for question = 7 marks)

Q15.

The frequency table shows information about the weights of 80 adults.

Weight (w kg)	Frequency
$40 < w \leqslant 50$	4
$50 < w \leqslant 60$	7
$60 < w \leqslant 70$	21
$70 < w \leqslant 80$	21
$80 < w \leqslant 90$	18
$90 < w \leqslant 100$	7
$100 < w \leqslant 110$	2

Weight (w kg)	Cumulative frequency
$40 < w \leqslant 50$	4
$40 < w \leqslant 60$	
$40 < w \leqslant 70$	
$40 < w \leqslant 80$	
$40 < w \leqslant 90$	
$40 < w \leqslant 100$	
$40 < w \leqslant 110$	

(a) Complete the cumulative frequency table.

(1)

(b) On the grid, draw a cumulative frequency graph for your table.

(2)
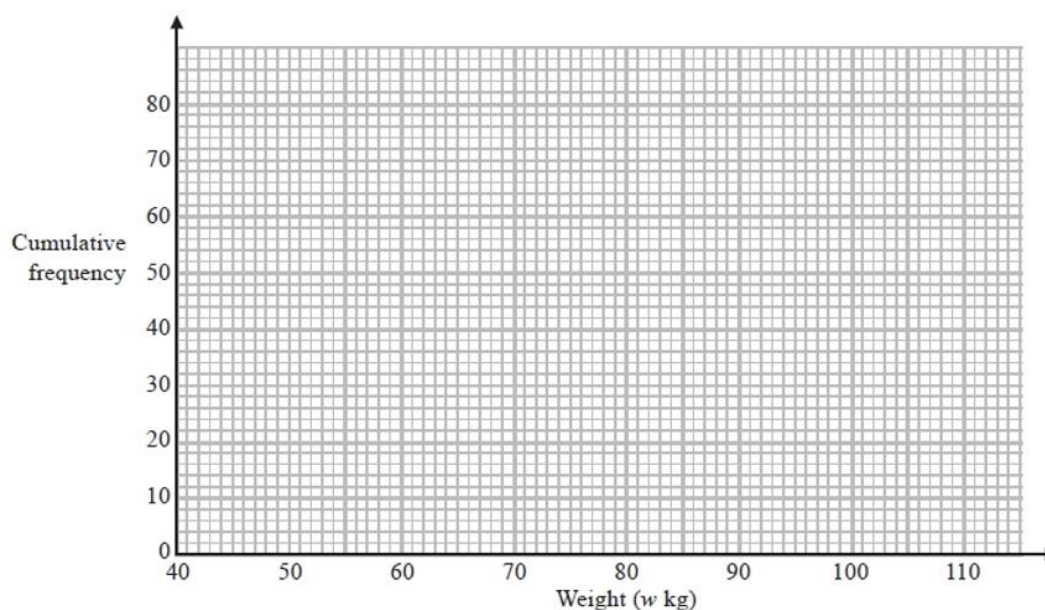
(c) Use your graph to find an estimate for the number of adults with weight more than 85 kg.

.... (2)

(d) Use your graph to find an estimate for the interquartile range of the weights of the adults.

.... kg (2)

(Total for question = 7 marks)

Q16.

T is directly proportional to the cube of r

$T = 21.76$ when $r = 4$

(a) Find a formula for T in terms of r

(3)

(b) Work out the value of T when $r = 6$

(1)

(Total for question = 4 marks)

_____

Q17.

The point A has coordinates $(0, 2)$
The point B has coordinates $(-4, -1)$

(a) Find the coordinates of the midpoint of AB.

………..…………………………… **(2)**

(b) Find the distance between the two points A and B.

………………………… **(2)**

(Total for question = 4 marks)
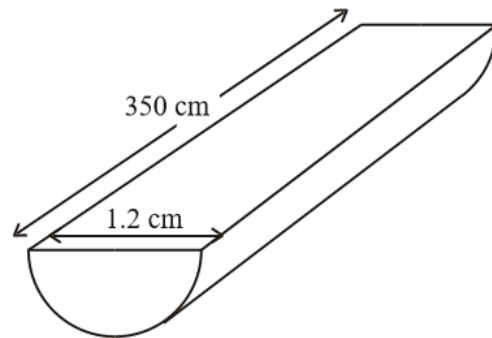
_____

Q18.

Diagram **NOT** accurately drawn

The diagram shows a piece of wood.
The piece of wood is a prism of length 350 cm.
The cross-section of the prism is a semi-circle with diameter 1.2 cm.

Calculate the volume of the piece of wood.
Give your answer correct to 3 significant figures.

(Total for question = 4 marks)

Q19.

Louise invests £x in Better Investments for 3 years.
Sadiq invests £x in County Bank for 3 years.

Better Investments	County Bank
Compound Interest	Compound Interest
2.5% per annum	2% per annum for the first two years 3.5% per annum for each extra year

At the end of the 3 years, the value of Louise's investment is £344 605

Work out the value of Sadiq's investment at the end of the 3 years.

_____

(Total for question = 4 marks)

Q20. Line **L** has equation $4y - 6x = 33$
Line **M** goes through the point A (5, 6) and the point B (−4, k)

L is perpendicular to **M**.

Work out the value of k.

(Total for question = 4 marks)

Q21.

Natalie makes potato cakes in a restaurant.

She mixes potato, cheese and onion so that

weight of potato : weight of cheese : weight of onion = 9 : 2 : 1

Natalie needs to make 6000 g of potato cakes.

Cheese costs £2.25 for 175 g.

Work out the cost of the cheese needed to make 6000 g of potato cakes.

(Total for question = 4 marks)

Q22.

- Andy cycles a distance of 30 km at an average speed of 24 km/h.
 He then runs a distance of 12 km at an average speed of 8 km/h.

 Work out the total time Andy takes.
 Give your answer in hours and minutes.

.... hours .... minutes

(Total for question is 3 marks)

_____

Q23.

Solve the simultaneous equations

$$5x - 9y = -46$$
$$y = -2x$$

x = ...
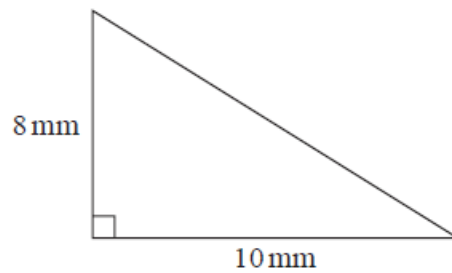
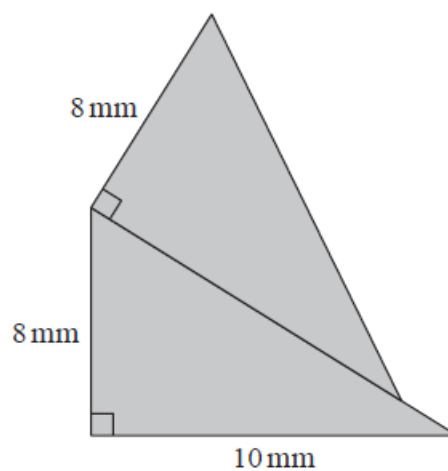y = ...

(Total for question = 3 marks)

Q24.

Here is a right-angled triangle.

8 mm

10 mm

The shaded shape below is made from two of these triangles.

8 mm

8 mm

10 mm

Work out the perimeter of the shaded shape.
Give your answer correct to 3 significant figures.

.... mm

(Total for question = 4 marks)

Q25.

Across 25 football matches, a football team has a mean score of 2.80 goals per match.
The team have one more match left in the tournament. They want to raise their mean number of goals per match to 3.00 for this tournament.

How many goals does the football team need to score in their final match to achieve this?

_____

(Total for question = 3 marks)

_____

END OF QUESTIONS

CHECK YOUR WORK

IF YOU HAVE TIME, TRY THE FOLLOWING:

1,   Five friends met for an afternoon of tennis. They played five sets of doubles, each set being played by a different group of four friends.

The sum of the ages of the players in each set was 124, 128, 130, 136 and 142 years, respectively.

How old was the youngest player?

2.   Emma walked to school and the trip usually took 20mins.

One day she realised that she'd left her homework behind. She worked out that if she continued she'd arrive at school 8mins before the bell went. If, however, she turned back and fetched her homework, she'd be 10mins late.

What fraction of the journey had she walked before realising that she had left her homework behind?

3.   If $z + \dfrac{1}{z} = 3$,   what is the value of $z^2 + \dfrac{1}{z^2}$?