Emanuel School

Entrance Exam

Sample Maths Paper

Sixth Form Entry

Time Allowed: 1 hour

Surname	
First name	

16+ Mathematics Practice Questions

Question 1

Make x the subject of the formula:

$$y = \sqrt{\frac{6x+8z}{-5}}$$

$x =$ .....

Question 2

Make a the subject of $\frac{3b}{7a} + 9d = 3c$

$a =$ .....

Question 3

Make x the subject of the formula:

$$4y = \frac{6x+2z}{3}$$

$x =$ .....

Question 4

Make a the subject of the formula where a is positive:

$a + 2b = 6ac + 5d$

$a =$ .....

Question 5

Make b the subject of $\frac{9}{a} = \frac{9}{b} - \frac{1}{c}$

$b =$ .....

Question 6

Make x the subject of the formula where x is positive:

$y = \sqrt{\frac{3x-6}{4x}}$

$x =$ .....

Question 7

Expand and simplify

$(3x-3)(x+5) - (2x+2)(3x+3)$

.....

Question 8

Expand and simplify:

$(x+3)^2$

.....

Question 9

Expand and simplify

$\left(3-\frac{3}{x}\right)^2$

.....

Question 10

Expand and simplify:

$(3 + 2x)^3 - (1 - 3x)^2$

.....

Question 11

Factorise fully

$6(p - 8) + 4(p - 8)^2$

.....

Question 12

Factorise completely

$13x^3y^3z^5 - 13x^5y^5z^3$

.....

Question 13

Factorise the following:

$64 - y^2$

.....

Question 14

Factorise:

$3x^2 - 11x - 4$

.....

Question 15

Factorise

$2x^2 - 7xy + 6y^2$

.....

Question 16

Evaluate $a^2 + \frac{12}{b}$ when $a = -4$ and $b = 6$

.....

Question 17

Simplify

$8x^2y^3 \times 3x$

.....

Question 18

Simplify

$8x \times 8z$

.....

Question 19

Simplify the expression below.

$26x^2z \div 13z$

.....

Question 20

Simplify

$\frac{10x^8y^8}{5x^7y^5}$

.....

Question 21

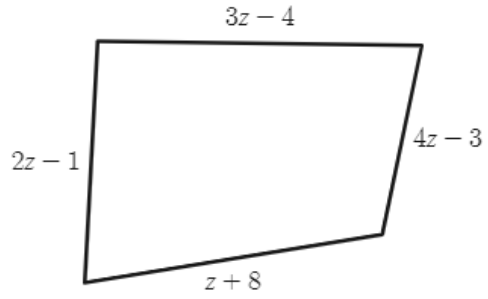


Diagram not drawn accurately.

Form an expression in terms of z for the perimeter of the shape.
Give your answer in its simplest form.

.....

Question 22

Solve the following simultaneous equations.

$$\begin{cases} 4x + y = 13 \\ y = 6x - 22 \end{cases}$$

$x = \ldots\ldots\ldots, y = \ldots\ldots\ldots$

Question 23

Solve the following simultaneous equations.

$$\begin{cases} 4x - 4y = -2 \\ 5x + 3y = 13.5 \end{cases}$$

$x = \ldots\ldots\ldots, y = \ldots\ldots\ldots$

Question 24

Solve the following simultaneous equations.

$$\begin{cases} x + 3y = 4 \\ x^2 + y^2 = 80 \end{cases}$$

$x = \ldots\ldots\ldots, y = \ldots\ldots\ldots$

or $x = \ldots\ldots\ldots, y = \ldots\ldots\ldots$

Question 25

Solve the following simultaneous equations.

$$\begin{cases} y = 3x + 5 \\ x^2 + y^2 = 5 \end{cases}$$

$x = \ldots\ldots\ldots, y = \ldots\ldots\ldots$

or $x = \ldots\ldots\ldots, y = \ldots\ldots\ldots$

Question 26

Solve:

$11 - 3f = -1$

$f = \ldots\ldots\ldots$

Question 27

Solve:

$2(2x + 8) - 2 = x - 10$

$x =$.....

Question 28

Solve for x:

$\frac{4}{5} = \frac{5x+3}{2}$

$x =$.....

Question 29

Solve:

$\frac{2x-6}{4} + \frac{6x+3}{5} = 6x + 6$

$x =$.....

Question 30

Solve:

$3x^2 - 8x + 4 = 0$

$x =$ .....

or $x =$ .....

Question 31

Solve

$(x + 6)(x + 8) = 0$

$x =$ .....

or $x =$ .....

Question 32

Find the gradient of the line passing through $(-1, -5)$ and $(-2, 4)$

.....

Question 33

A line has equation $3x + 2y = 2$

Find the y-intercept.

y-intercept = ...

Question 34

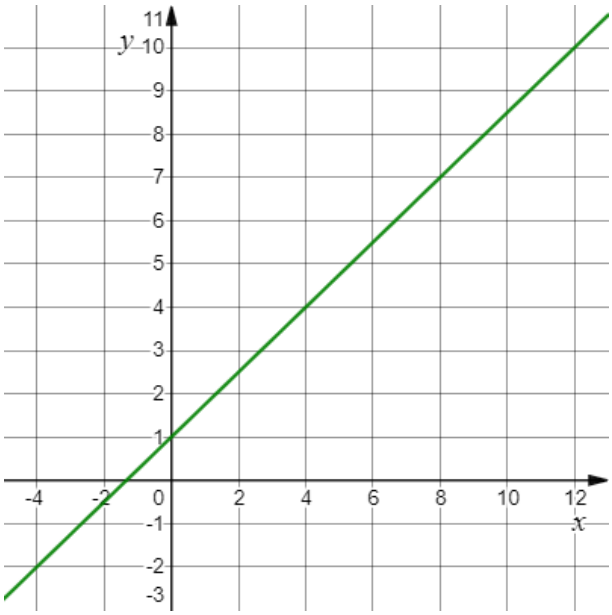
A line has equation $y = \frac{1}{2}x + 5$

Find the gradient.

gradient = ...

Question 35

Find an equation of the line drawn below.



.....

Question 36

Find an equation of the line with gradient -1 and that passes through the point $(4, -5)$

.....

Question 37

Find an equation of the line parallel to $y = 3x + 3$ and that passes through the point $(-2, 4)$

.....

Question 38

Find an equation of the line perpendicular to $y = -2x + 1$ and that passes through the point $(-3, -2)$

.....

Mark scheme

Question 1

$\frac{-5y^2-8z}{6}$

$$y = \sqrt{\frac{6x+8z}{-5}}$$
$$y^2 = \frac{6x+8z}{-5}$$
$\times -5 \downarrow$ $\downarrow \times -5$
$$-5y^2 = 6x + 8z$$
$-8z \downarrow$ $\downarrow -8z$
$$-5y^2 - 8z = 6x$$
$\div 6 \downarrow$ $\downarrow \div 6$
$$\frac{-5y^2-8z}{6} = x$$

$\therefore\ x = \frac{-5y^2-8z}{6}$

Question 2

$\frac{3b}{7(3c-9d)}$

① Make a the subject.

$$\frac{3b}{7a} + 9d = 3c$$
$-9d \downarrow$ $\downarrow -9d$
$$\frac{3b}{7a} = 3c - 9d$$
$\times 7a \downarrow$ $\downarrow \times 7a$
$$3b = 7a(3c - 9d)$$
$\div 7(3c-9d) \downarrow$ $\downarrow \div 7(3c-9d)$
$$\frac{3b}{7(3c-9d)} = a$$

Question 3

$\frac{12y-2z}{6}$

$$4y = \frac{6x+2z}{3}$$
$\times 3 \downarrow$ $\downarrow \times 3$
$$12y = 6x + 2z$$
$-2z \downarrow$ $\downarrow -2z$
$$12y - 2z = 6x$$
$\div 6 \downarrow$ $\downarrow \div 6$
$$\frac{12y-2z}{6} = x$$

$\therefore\ x = \frac{12y-2z}{6}$

Question 4

$\frac{5d-2b}{1-6c}$

You need to expand the bracket, put a on the left hand-side, factorise by a, and then divide by the bracket.

$$\begin{aligned} a + 2b &= 6ac + 5d \\ a - 6ac &= 5d - 2b \\ a(1-6c) &= 5d - 2b \\ \div(1-6c)\downarrow \quad & \quad \downarrow \div(1-6c) \\ a &= \frac{5d-2b}{1-6c} \end{aligned}$$

$\therefore\ a = \frac{5d-2b}{1-6c}$

Question 5

$\frac{9ac}{9c+a}$

① Write the right-hand side as a single fraction.

$$\begin{aligned} \frac{9}{a} &= \frac{9}{b} - \frac{1}{c} \\ \frac{9}{a} &= \frac{9c}{bc} - \frac{b}{bc} \\ \frac{9}{a} &= \frac{9c-b}{bc} \end{aligned}$$

② Cross-multiply.

$$\begin{aligned} 9bc &= a(9c-b) \\ 9bc &= 9ac - ab \end{aligned}$$

③ Isolate b.

$$\begin{aligned} 9bc + ab &= 9ac \\ b(9c+a) &= 9ac \\ b &= \frac{9ac}{9c+a} \end{aligned}$$

Question 6

$\frac{-6}{4y^2-3}$

You need to square both sides, multiply by the denominator, put x on the left hand-side, factorise by x, and then divide by the bracket.

$$y = \sqrt{\frac{3x-6}{4x}}$$
$$y^2 = \frac{3x-6}{4x}$$
$$\times 4x \downarrow \quad \downarrow \times 4x$$
$$4y^2x = 3x - 6$$
$$-3x \downarrow \quad \downarrow -3x$$
$$4y^2x - 3x = -6$$
$$x(4y^2 - 3) = -6$$
$$\div (4y^2-3) \downarrow \quad \downarrow \div (4y^2-3)$$
$$x = \frac{-6}{4y^2-3}$$

$\therefore\ x = \frac{-6}{4y^2-3}$

Question 7

$-3x^2 - 21$

① Expand $(3x - 3)(x + 5)$

$$(-)(+)$$
$$= \times + \times - \times - \times$$
$$= 3x^2 + 12x - 15$$

② Expand $(2x + 2)(3x + 3)$

$$(+)(+)$$
$$= \times + \times + \times + \times$$
$$= 6x^2 + 12x + 6$$

③ Simplify.

$$(3x - 3)(x + 5) - (2x + 2)(3x + 3)$$
$$= \left(3x^2 + 12x - 15\right) - \left(6x^2 + 12x + 6\right)$$
$$= -3x^2 - 21$$

Question 8

$x^2 + 6x + 9$

① Multiply each term in the first bracket by each term in the second bracket.

$$(x + 3)^2$$
$$= (+)(+)$$
$$= \times + \times + \times + \times$$
$$= x^2 + 3x + 3x + 9$$

② Simplify.

$= x^2 + 6x + 9$

Question 9

$9 - \frac{18}{x} + \frac{9}{x^2}$

① Expand the brackets and simplify.

$(-)(-)$
$= \times - \times - \times + \times$
$= 9 - \frac{9}{x} - \frac{9}{x} + \frac{9}{x^2}$
$= 9 - \frac{18}{x} + \frac{9}{x^2}$

Question 10

$8x^3 + 27x^2 + 60x + 26$

① Write the cubed term as the product of 3 brackets and expand the first two.

$(3 + 2x)\ 3$
$= (3 + 2x)\,(3 + 2x)\,(3 + 2x)$
$= \left(9 + 6x + 6x + 4x^2\right)(3 + 2x)$
$= \left(9 + 12x + 4x^2\right)(3 + 2x)$

② Expand the remaining two brackets and simplify this expression.

$= 27 + 36x + 12x^2 + 18x + 24x^2 + 8x^3$
$=$

③ Write the squared term as the product of 2 brackets and expand them.

$(1 - 3x)\ 2$
$= (1 - 3x)\,(1 - 3x)$
$= 1 - 3x - 3x + 9x^2$
$=$

④ Subtract the second expanded expression from the first.

$(3 + 2x)^3 - (1 - 3x)^2$
$= \ -$
$= 8x^3 + 27x^2 + 60x + 26$

Question 11

$2(p - 8)(2p - 13)$

① Identify the common factors.

$$\equiv 3 \times 2(p-8) + 2 \times 2(p-8)(p-8)$$
$$\equiv 2(p-8)(3+2(p-8))$$

② Simplify.

$$\equiv 2(p-8)(2p-13)$$

Question 12

$13x^3y^3z^3(z^2-x^2y^2)$

You need to find the highest number that is a common factor of 13 and -13. This number is 13.
For the variables, take the lowest power of each variable in common, so $x^3y^3z^3$

We can write $13x^3y^3z^3(\quad)$ and consider what we need to multiply $13x^3y^3z^3$ by to get each of the terms.

$13x^3y^3z^5-13x^5y^5z^3 = 13x^3y^3z^3(z^2-x^2y^2)$

Question 13

$(8+y)\,(8-y)$

&#9312**Apply difference of two squares method:**We know that $(a+b)(a-b) \equiv a^2-b^2$We can reverse this process:(Could not display math)This method is known as **the difference of two squares.**

Question 14

$(3x + 1)(x-4)$

You need to find 2 numbers that add to give -11 and multiply to give $3 \times -4 = -12$.

$\oplus \ -11 \ \otimes \ -12$

The two numbers are -12 and 1.

Split the middle term $-11x$ as $-12x + x$ and then factorise the first two terms and last two terms.

$$\begin{aligned} 3x^2-11x-4 &= 3x^2-12x+x-4 \\ &= 3x(x-4) + 1(x-4) \\ &= (3x+1)(x-4) \end{aligned}$$

Question 15

$(2x-3y)(x-2y)$

① Consider $2x^2 - 7x + 6$ and factorise.

$$2x^2 - 7x + 6 \ = \ (2x - 3)(x - 2)$$

② Adjust the brackets to obtain the correct expansion.

$$2x^2 - 7xy + 6y^2 \ = \ (2x - 3y)(x - 2y)$$

Question 16

18

① Substitute $a \ = \ -4$ and $b \ = \ 6$

$$\begin{aligned} a^2 + \frac{12}{b} \ &= \ (-4)^2 + \frac{12}{6} \\ &= \ 16 + 2 \\ &= \ 18 \end{aligned}$$

Question 17

$24x^3y^3$

You can start by multiplying the coefficients: $8 \times 3 = 24$

x is multiplied by itself 2 times and 1 time so 3 times in total, therefore we have x^3

y is multiplied by itself 3 times and 0 time so 3 times in total, therefore we have y^3

Question 18

$64xz$

① Multiply the coefficients and variables.

$$8x \times \ 8z \ = \ 64xz$$

Question 19

$2x^2$

① Rewrite the expression as a fraction.

$$26x^2z \div \ 13z = \frac{26x^2z}{13z}$$

② Cancel out common factors in the numerator and denominator then simplify.

$$\frac{\cancel{2}\cancel{6}^{2}x^{2}z}{\cancel{1}\cancel{3}^{1}z} = 2x^{2}$$

Question 20

$2xy^{3}$

① Divide the coefficients then the variables.

$$\frac{10x^{8}y^{8}}{5x^{7}y^{5}} = xy$$

Question 21

$10z$

The perimeter is the sum of all the sides.

$$(z+8) + (4z-3) + (3z-4) + (2z-1)$$
$$= 10z$$

Question 22

$x = 3.5, y = -1$

① Substitute the second equation into the first one.

$$\begin{aligned} 4x + 1(6x-22) &= 13 \\ 4x + 6x - 22 &= 13 \\ 10x - 22 &= 13 \\ 10x &= 35 \\ x &= 3.5 \end{aligned}$$

② Substitute $x = 3.5$ into the second equation to find y.

$$\begin{aligned} y &= 6 \times 3.5 - 22 \\ &= -1 \end{aligned}$$

Question 23

$x = 1.5, y = 2$

Label the equations.

$$\begin{cases} 4x - 4y = -2 \ ① \\ 5x + 3y = 13.5 \ ② \end{cases}$$

Multiply ① by 3 and ② by 4.

$$\begin{cases} 12x - 12y = -6 \ ① \\ 20x + 12y = 54 \ ② \end{cases}$$

Do ① + ② then solve for x.

$$\begin{aligned} 32x &= 48 \\ x &= \frac{3}{2} \end{aligned}$$

Substitute $x = \frac{3}{2}$ into ① and solve for y.

$$\begin{aligned} 4(\tfrac{3}{2}) - 4y &= -2 \\ 6 - 4y &= -2 \\ -4y &= -8 \\ y &= 2 \end{aligned}$$

Question 24

$x = \frac{44}{5}, y = -\frac{8}{5}$ or $x = -8, y = 4$

Rearrange $x + 3y = 4$ to make y the subject.

$$\begin{aligned} x + 3y &= 4 \\ 3y &= 4 - x \\ y &= \frac{4-x}{3} \end{aligned}$$

Substitute $y = \frac{4-x}{3}$ into $x^2 + y^2 = 80$ then solve.

$$\begin{aligned} x^2 + \left(\frac{4-x}{3}\right)^2 &= 80 \\ x^2 + \frac{(4-x)(4-x)}{9} &= 80 \\ 9x^2 + (4-x)(4-x) &= 720 \\ 9x^2 + 16 - 4x - 4x + x^2 &= 720 \\ 10x^2 - 8x + 16 &= 720 \\ 10x^2 - 8x - 704 &= 0 \\ \therefore\ x = \frac{44}{5} \text{ or } x &= -8 \end{aligned}$$

Substitute these values into $x + 3y = 4$

When $x = \frac{44}{5}$, $1\left(\frac{44}{5}\right) + 3y = 4$

$$y = -\frac{8}{5}$$

When $x = -8$, $1(-8) + 3y = 4$

$$y = 4$$

Question 25

$x = -1, y = 2$ or $x = -2, y = -1$

Substitute $y = 3x + 5$ into $x^2 + y^2 = 5$ then solve.

$$\begin{aligned} x^2 + (3x+5)^2 &= 5 \\ x^2 + (3x+5)(3x+5) &= 5 \\ x^2 + 9x^2 + 15x + 15x + 25 &= 5 \\ 10x^2 + 30x + 25 &= 5 \\ 10x^2 + 30x + 20 &= 0 \\ x^2 + 3x + 2 &= 0 \\ (x+1)(x+2) &= 0 \\ \therefore\ x = -1 \text{ or } x &= -2 \end{aligned}$$

Substitute these values into $y = 3x + 5$

$$\begin{aligned} \text{When } x = -1,\ y &= 3(-1) + 5 \\ &= 2 \end{aligned}$$

$$\begin{aligned} \text{When } x = -2,\ y &= 3(-2) + 5 \\ &= -1 \end{aligned}$$

Question 26

$f = 4$

We change the equation by doing the same thing to each side with the goal of making it simpler at each step.

Firstly, we remove the constant from the left hand side by subtracting 11 from both sides and then we divide by the coefficient of the f-term on the left hand side, which is -3.

$$\begin{aligned} 11 - 3f &= -1 \\ -11 \downarrow \quad &\downarrow -11 \\ -3f &= -12 \\ \div -3 \downarrow \quad &\downarrow \div -3 \\ f &= 4 \end{aligned}$$

Question 27

$x = -8$

We change the equation by doing the same thing to each side with the goal of making it simpler at each step.

Firstly, we expand the bracket on the left hand side and simplify the resulting expression. Next we remove the x-term from one of the sides. We pick the right hand side because it has the smaller x-term, so we subtract x from both sides. Then we remove the constant

from the left hand side by subtracting 14 from both sides. Lastly, we divide by the coefficient of the x-term on the left hand side, which is 3.

$$\begin{aligned}
2(2x + 8) - 2 &= x - 10 \\
4x + 16 - 2 &= x - 10 \\
4x + 14 &= x - 10 \\
-x \downarrow \quad &\downarrow -x \\
3x + 14 &= -10 \\
-14 \downarrow \quad &\downarrow -14 \\
3x &= -24 \\
\div 3 \downarrow \quad &\downarrow \div 3 \\
x &= -8
\end{aligned}$$

Question 28

$$x = -\frac{7}{25}$$

When a fraction equals another, you can cross-multiply, then isolate x and solve.

$$\begin{aligned}
\frac{4}{5} &= \frac{5x+3}{2} \\
8 &= 25x + 15 \\
-15 \downarrow \quad &\downarrow -15 \\
-7 &= 25x \\
\div 25 \downarrow \quad &\downarrow \div 25 \\
-\frac{7}{25} &= x
\end{aligned}$$

Question 29

$$x = -\frac{69}{43}$$

The lowest common denominator is 20 so we can multiply the first fraction top and bottom by 5 and the second fraction top and bottom by 4.

$$\begin{aligned}
\frac{2x-6}{4} + \frac{6x+3}{5} &= 6x + 6 \\
\frac{10x-30}{20} + \frac{24x+12}{20} &= 6x + 6 \\
\frac{10x-30+24x+12}{20} &= 6x + 6 \\
\frac{34x-18}{20} &= 6x + 6 \\
\times 20 \downarrow \quad &\downarrow \times 20 \\
34x - 18 &= 120x + 120 \\
-34x \downarrow \quad &\downarrow -34x \\
-18 &= 86x + 120 \\
-120 \downarrow \quad &\downarrow -120 \\
-138 &= 86x \\
\div 86 \downarrow \quad &\downarrow \div 86 \\
-\frac{69}{43} &= x
\end{aligned}$$

Question 30

$x = \frac{2}{3}$ or $x = 2$

① Factorise.

$$3x^2 - 8x + 4 = 0$$
$$(3x - 2)(x - 2) = 0$$

② Write the first factor equal to zero and solve.

$$(3x - 2)\ (x - 2) = 0$$

$$3x - 2 = 0$$
$$3x = 2$$
$$x = \frac{2}{3}$$

③ Write the second factor equal to zero and solve.

$$(3x - 2)\ (x - 2) = 0$$

$$x - 2 = 0$$
$$x = 2$$

④ Conclude.

$x = \frac{2}{3}$ or $x = 2$

Question 31

$x = -6$ or $x = -8$

① Write the first factor equal to zero and solve.

$$(x + 6)\ (x + 8) = 0$$

$$x + 6 = 0$$
$$x = -6$$

② Write the second factor equal to zero and solve.

$$(x + 6)\ (x + 8) = 0$$

$$x + 8 = 0$$
$$x = -8$$

③ Conclude.

$x = -6$ or $x = -8$

Question 32

-9

The gradient of any straight line is $m = \frac{\text{change in } y}{\text{change in } x}$

$$m = \frac{4 - -5}{-2 - -1}$$
$$= -9$$

Question 33

1

An equation of a straight line can be written in the form $y = mx + c$ where m is the gradient and c is the y-intercept.

$$3x + 2y = 2$$
$$2y = -3x + 2$$
$$y = -\frac{3}{2}x + 1$$

∴ the y-intercept is $c = 1$

Question 34

$\frac{1}{2}$

An equation of a straight line can be written in the form $y = mx + c$ where m is the gradient and c is the y-intercept.

∴ the gradient is $m = \frac{1}{2}$

Question 35

$y = \frac{3}{4}x + 1$

An equation of a straight line can be written in the form $y = mx + c$ where m is the gradient and c is the y-intercept.

$m = \frac{\text{change in } y}{\text{change in } x}$

Taking the two opposite ends of the line:

$$m = \frac{10 - -2}{12 - -4}$$
$$= \frac{12}{16}$$
$$= \frac{3}{4}$$

The y-intercept of the line is where the line crosses the y-axis. Here the y-intercept is 1

The equation of the line is $y = \frac{3}{4}x + 1$

Question 36

$y = -x - 1$

① Substitute $m = -1$ in $y = mx + c$.

$$y = -1x + c$$

② Substitute $x = 4$ and $y = -5$ in $y = -1x + c$.

$$-5 = -1 \times 4 + c$$
$$-5 = -4 + c$$
$$c = -5 + 4$$
$$= -1$$

③ Conclude.

$$y = -x - 1$$

Question 37

$y = 3x + 10$

Parallel lines have same gradients, so the gradient of the line is 3

The equation can be written in the form $y = mx + c$ where m is the gradient and c is the y-intercept.

Substituting the gradient gives $y = 3x + c$

Substituting $x = -2$ and $y = 4$ give $4 = 3 \times -2 + c$

Rearranging gives $c = 4 - 3 \times -2 = 10$

Therefore the equation is $y = 3x + 10$

Question 38

$y = \frac{1}{2}x - \frac{1}{2}$

① Find the gradient of the line

$m = -2$

② Find the negative reciprocal of $m = -2$

$m_{\perp} = \frac{1}{2}$

③ Substitute $m = \frac{1}{2}$, $x = -3$ and $y = -2$ into $y = mx + c$

$$-2 = \frac{1}{2} \times -3 + c$$
$$c = -2 - \frac{1}{2} \times -3$$
$$= -\frac{1}{2}$$

④ Substitute $m = \frac{1}{2}$ and $c = -\frac{1}{2}$ into $y = mx + c$

$y = \frac{1}{2}x - \frac{1}{2}$
