

Eton College King's Scholarship Examination 2020

MATHEMATICS B

(One and a half hours)

Candidate number:

Please write your candidate number on EVERY sheet.

Please answer on the paper in the spaces provided.

There are 8 questions: each one is worth 10 marks.

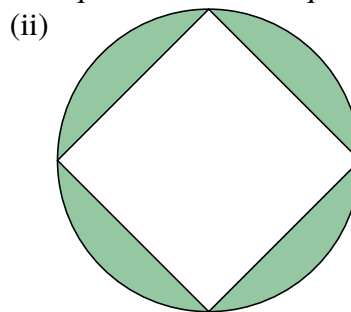
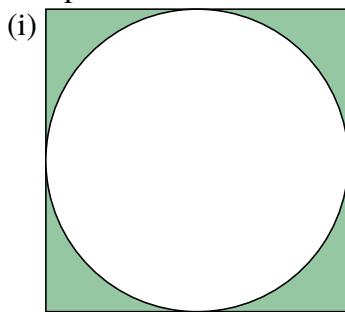
Calculators are allowed, but you should show all your working.

Do not turn over until told to do so.

1. (a) The product of the length of the diagonals of a square is 450 cm^2 .
Find the area of the square.

[2]

- (b) The pictures show a round plug in a square hole and a square plug in a round hole.



In both pictures, the circle has radius r .

Find the ratio of the shaded area in (i) to the shaded area in (ii). Give your answer in the form $1 : x$, where x is a decimal to 3 significant figures. [4]

- (c) Two circles of radius 10 cm and 24 cm have centres at A and C respectively. The circles intersect at the points B and D and the distance AC is 26 cm.

By considering the relationship between AB, BC and AC, find the area of the quadrilateral ABCD.

[4]

2. (a) To make 'short' pastry one uses flour to fat in the ratio 2 : 1. To make 'flaky' pastry requires a ratio of 4 : 3.

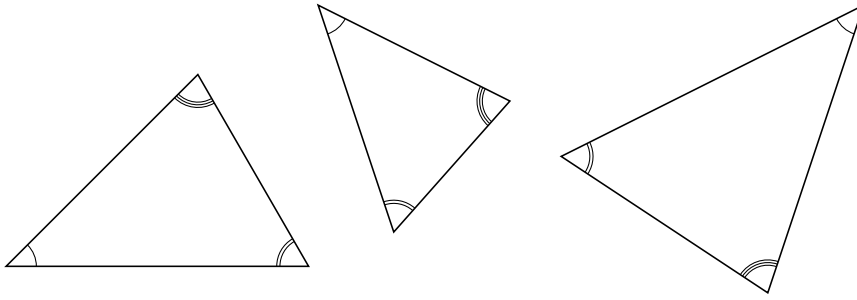
I have 29 kg of flour and 20 kg of fat and wish to use all the ingredients making some of each type of pastry. How much flaky pastry do I make? [5]

- (b) By weight, raspberries are 85% water, raspberry jam is 30% water, and sugar contains no water. I make raspberry jam by mixing equal weights of raspberries and sugar and then boiling them to evaporate off some of the water.

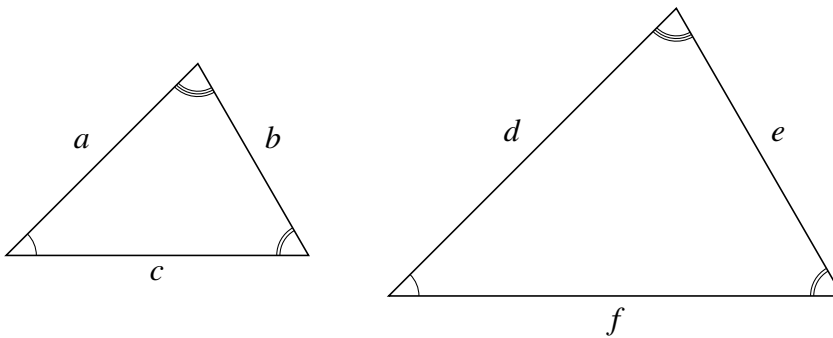
What weight of jam can I make with 2.8 kg of raspberries?

[5]

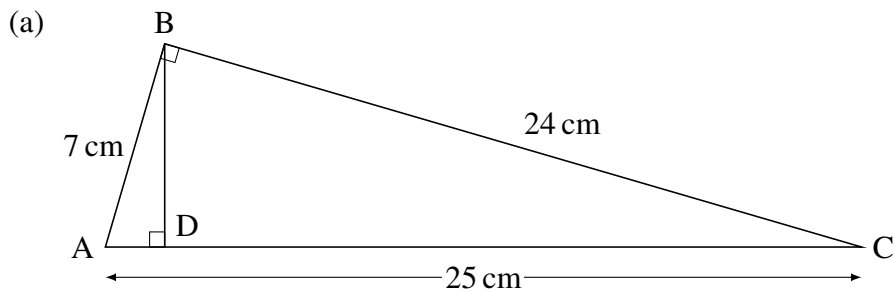
3. Two triangles are said to be *similar* if the angles in one are equal to the angles in the other. The three triangles below are all similar.



Similar triangles' sides are in the same ratios.



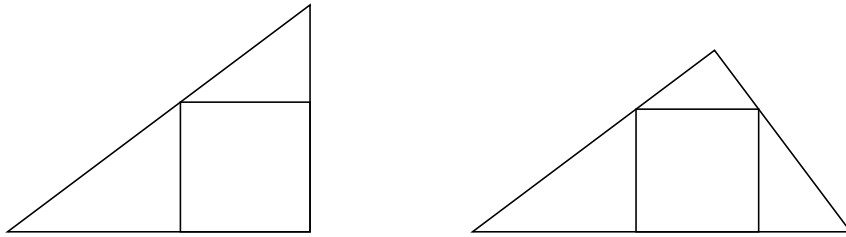
$$\frac{a}{d} = \frac{b}{e} = \frac{c}{f}$$



- (i) By comparing angles show that $\triangle ABC$ and $\triangle ADB$ are similar. [2]

- (ii) Using the similar triangles facts, find length BD. [2]

- (b) There are two different inscribed squares that can be drawn in a triangle with side lengths of 3 cm, 4 cm and 5 cm.



Show that the ratio of the side lengths of the squares is 37 : 35

[6]

4. On the island of Etonia, I met three people A, B, and C, one of whom is a Teacher, one is an Oppidan, and the other is a Scholar. The Teacher always tells the truth, the Oppidan always lies, and the Scholar can either lie or tell the truth. (a), (b), (c) and (d) are separate questions.

(a) A says, 'I am the Teacher.'

B says, 'I am the Oppidan.'

C says, 'I am the Scholar.'

Who is the Scholar? Briefly explain your answer.

[3]

(b) Suppose instead that A says 'I am the Oppidan' and B says 'Yes, that's true, A is the Oppidan'. (C says nothing.)

Who is the Teacher? Who is the Oppidan? Who is the Scholar? Briefly explain your answer.

[2]

- (c) Suppose instead that C says 'B is the Teacher' and B says 'That's wrong'. (A says nothing.)

Who is the Teacher? Who is the Oppidan? Who is the Scholar? Briefly explain your answer.

[2]

- (d) Suppose instead that A says 'B is the Scholar' and C says 'A is the Oppidan'. Then B says 'You have heard enough to determine who the Teacher is'.

Who is the Teacher? Who is the Oppidan? Who is the Scholar? Briefly explain your answer.

[3]

5. (a) Solve for p , q and r .

[3]

$$p + q = 13$$

$$q + r = 37$$

$$r + p = 15$$

(b) Solve for positive numbers, x , y and z .

[3]

$$xy = 1125$$

$$yz = 864$$

$$xz = 750$$

(c) Solve for positive numbers a , b and c .

[4]

$$a(b + c) = 120$$

$$b(c + a) = 144$$

$$c(a + b) = 168$$

6. (a) A man and a son are comparing their ages. The father says 'Your age now is the same as my age written backwards'. The son says 'Last year you were twice as old as I was then'. How old are they now? [3]

- (b) I have 20 iron bars. Some are 3 kg, some are 8 kg and the rest are 14 kg. The total mass of all twenty is 183 kg.

Determine the number of each bar.

[7]

7. The mean of two positive numbers, a and b , is $\frac{a+b}{2}$

This is properly called the *arithmetic mean* (AM) and is actually just one of many different types of means. Two others are:

Geometric mean (GM): \sqrt{ab}

Harmonic mean (HM): $\frac{2ab}{a+b}$

- (a) Show that in each case $a < \text{HM} < \text{GM} < \text{AM} < b$

(i) $a = 4, b = 9$ [1]

(ii) $a = 50, b = 800$ [1]

- (b) (i) Explain why, for any numbers a and b , $(a - b)^2 \geq 0$ [1]

(ii) Hence show that $2ab \leq a^2 + b^2$

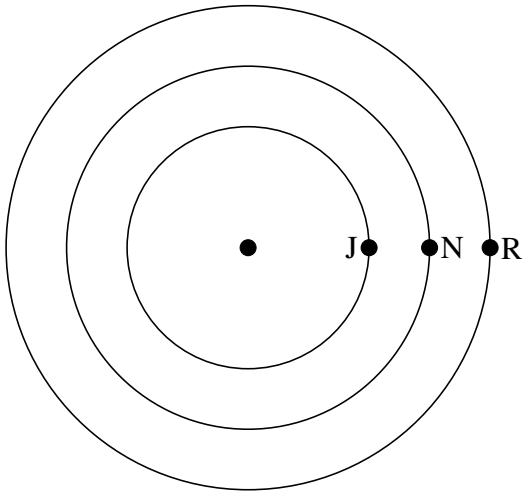
[2]

(c) Hence show that for positive numbers x and y , $\sqrt{xy} \leq \frac{x+y}{2}$ [2]

(d) Show further that, for positive x and y , $\frac{2xy}{x+y} \leq \sqrt{xy}$ [2]

(e) Under what circumstances is $AM = GM = HM$? [1]

8. Joel, Nick and Robin all run laps of a circular track at constant speeds. Joel has the inside lane and takes 1.25 minutes to finish a lap, Nick takes 3 minutes to finish a lap and Robin takes 10 minutes to finish a lap. They all start lined up across the track, as shown, and run anticlockwise.



- (a) Show that it takes half an hour until all three runners are **next** at the starting point together. [2]

- (b) Show that there is an earlier time when all three runners are first on a straight line through the centre with at least one at the starting point and at least one on the opposite side. State how many laps each runner will have run at this time. [2]

- (c) Find the first time after the start that all three runners form a straight line with the centre of the track (not necessarily in their original positions nor on the same side of the centre as each other). Give the time in minutes and seconds, to the nearest second. [6]