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Eton College 13+ Maths 2021

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Eton College King's Scholarship Examination 2021

MATHEMATICS A

(One and a half hours)

Candidate Number:.....

Please write your candidate number on EVERY sheet.

Please answer on the paper in the spaces provided.

This paper is divided into two sections:

Section I (Short-answer questions) – 50 marks available

Section II (Extended questions) – 50 marks available

Answer all of Section I and as many questions as you can from Section II.

The marks for each part of each question are given in square brackets.

Show all your working.

No diagram is drawn to scale.

Neither calculators nor protractors may be used.

ADDITIONAL MATERIALS: NONE

Do not turn over until told to do so.

Section I: Short-answer questions (50 marks)

1. Find the value of the following, giving your answers as **reduced, mixed** fractions:

a) $\left(3\frac{2}{3} + \frac{2}{9}\right) \times 7\frac{2}{7}$ [3]

b) $\left(\frac{68}{19} \div \frac{17}{76}\right) \div \frac{6}{7}$ [4]

c) $327\frac{7}{12} + 271\frac{5}{9}$ [3]

d) $\left(4 - \frac{3}{4}\right)^2$ [3]

[Total for Question: 13]

2. Find the value of the following, giving your answer as a decimal:

a) 0.035×0.0022 [3]

b) $0.51 \div 0.068$ [3]

c) $(-1.1)^3$ [3]

[Total for Question: 9]

3. If $a = 1$ and $b = -2$, find the value of the following expressions, leaving your answers in simplified form:

a) $\frac{a}{b} - \frac{b}{a}$ [1]

b) $\frac{a^2+b^2}{a+b}$ [2]

[Total for Question: 3]

4. Simplify the following algebraic expressions fully, leaving no brackets in your final answers:

a) $2x - (3y + x) + \{3x - (5y - 4x + 7y)\}$ [2]

b) $a - [a - b - \{d - c + (a - b + c - d)\}]$ [2]

[Total for Question: 4]

5. Solve the following inequality, giving your final answer as a reduced, mixed fraction. In your final answer, x must appear on the left-hand side.

$$3 - 7x < 19 - 2x$$

[3]

[Total for Question: 3]

6. I have a glass containing five and two fifteenths fluid ounces of wine. I pour out one and seven twelfths fluid ounces of the wine. Find the volume of wine remaining in the glass, in fluid ounces as a reduced, **mixed** fraction: [4]

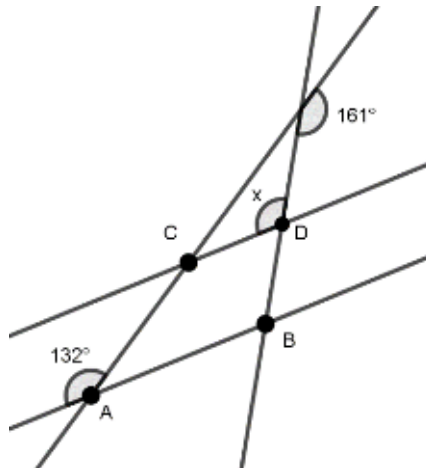
[Total for Question: 4]

7. Six years ago, Alice was 5 times as old as Beatrice was, but now she is only twice as old. Find the difference between the ages of Alice and Beatrice. [4]

[Total for Question: 4]

8. In the following diagram, line segments AB and CD are parallel. Calculate angle x .

[3]



[Total for Question: 3]

9. A class contains 30 pupils. 14 are boys and the rest are girls. In a test, the average mark of the boys is 62%, and the average mark of the girls is 68%. Find the average mark of the entire class, leaving your answer as a percentage correct to 1 decimal place.

[4]

[Total for Question: 4]

10. Solve the following equation, simplifying your final answer:

$$\frac{2}{3}\left(\frac{2x}{3} - 3\right) - \frac{1}{6}\left(\frac{3x}{2} - 8\right) = \frac{x}{12}$$

[3]

[Total for Question: 3]

Section II: Extended-answer questions (50 marks)

11. a) If one builder can build a wall in 5 hours, and a second builder can build one of the same size in 7 hours, how long will they take to build a wall working together? Give your answer in hours and minutes. [2]

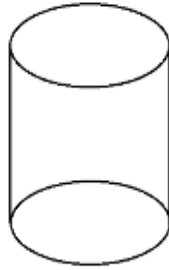
- b) If one builder can build a wall in A hours, and a second builder can build one of the same size in B hours, how many hours will they take to build a wall working together? Leave your answer as a single fraction. [2]

- c) If p litres of paint are required to paint a rectangular wall of side lengths $5q$ by q , how many litres of paint are required to paint a rectangular wall of side lengths $3r$ by r ? [3]

- d) A car travels at a rate of x feet in y seconds. How many hours does it take to travel z miles? You are given that 1 mile = 5280 feet. Leave your answer as a reduced, mixed fraction. [3]

[Total for Question: 10]

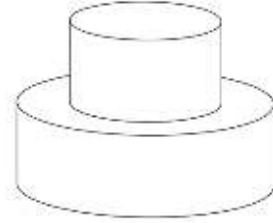
12. The volume of a cylinder is equal to the area of its circular base multiplied by its perpendicular height.



- a) Suppose a cylinder has eight times the perpendicular height of a second cylinder and has a circular base one tenth the diameter of the second cylinder. What is the ratio of the volume of the two cylinders? [3]

- b) Given that a suitably sized rectangular piece of paper can be wrapped around the curved surface of a cylinder so as to cover it exactly once with no overlap, write down a formula for the curved surface area of a cylinder in terms of the radius r of its base and its perpendicular height h . [2]

- c) A child's toy is made of two solid cylinders joined together as illustrated. The larger cylinder has diameter $6p$ cm and perpendicular height $2p$ cm. The smaller cylinder has diameter $4p$ cm and perpendicular height $2p$ cm. Find a formula for the total exposed surface area of the toy, leaving your answer simplified and in terms of p and π . [5]



[Total for Question: 10]

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13. Suppose that $x = 7.5\dot{3} = 7.53333 \dots$

a) i) Write down the value of $10x$ and $100x$. [2]

ii) Hence, prove that $90x = 678$ and deduce that $7.5\dot{3} = 7\frac{8}{15}$ [2]

Suppose now that $y = 1.\dot{9} = 1.9999 \dots$

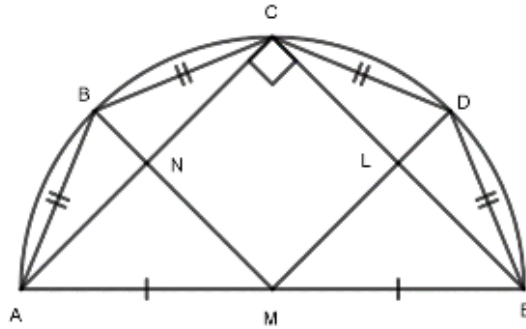
b) Using a method similar to part a), prove that $y = 2$. [2]

Suppose that $z = 17.\dot{b}c = 17.bcccc \dots$, where the letters b and c represent digits between 0 and 9 occurring in the normal decimal representation of a number.

- c) Find whole numbers u , v and w such that $z = \frac{u+vb+c}{w}$ [4]

[Total for Question: 10]

14. In the diagram, length AE equals $2 \times \sqrt{2}$ units and M is the mid-point of AE . Points B , C and D lie on a semicircle with diameter AE . Lengths AB , BC , CD and DE are all equal and angle ACE is a right angle.



a) Show that length AC equals 2 units. [1]

b) Prove that $CLMN$ is a square. [2]

c) Show carefully that length DL equals $\sqrt{2} - 1$ units. [2]

d) Show that length DE equals $\sqrt{4 - 2\sqrt{2}}$ units. [3]

e) Hence, prove that $\pi > 4 \times \sqrt{2 - \sqrt{2}}$. [2]

[Total for Question: 10]

15. *The Towers of Hanoi* is a puzzle consisting of three poles, labelled A, B and C, onto which punctured wooden discs of different sizes can be slid. Only one disc can be moved at a time, and at each stage every disc must be positioned so as to be smaller than the disc (if there is one) immediately beneath it. The object of the puzzle is to move the entire stack of discs, initially arranged vertically in order of decreasing size on pole A, to finish up arranged vertically in order of decreasing size on pole C.



- a) For the first round, the game is played with two discs only. The smaller disc is labelled 1, and the larger disc 2. Complete the table below illustrating the shortest possible method of finishing the puzzle.

Stage	Pole A Lowest position → Highest		Pole B Lowest position → Highest		Pole C Lowest position → Highest	
	Initial Position	2	1			
Move One	2		1			
Move Two						
Move Three					2	1

[1]

- b) For the second round, the game is played with three discs, labelled 1, 2 and 3 in order of increasing size. Complete the table below illustrating the shortest possible method of finishing the puzzle.

Stage	Pole A Lowest position → Highest			Pole B Lowest position → Highest			Pole C Lowest position → Highest		
	Initial Position	3	2	1					
Move One	3	2					1		
Move Two	3			2			1		
Move Three				2	1				
Move Four							3		
Move Five									
Move Six									
Move Seven							3	2	1

[3]

- c) For the third round, the game is played with four discs. By completing the table below, show the smallest number of steps necessary to complete the puzzle is 15.

Stage	Pole A Lowest position → Highest				Pole B Lowest position → Highest				Pole C Lowest position → Highest			
	Initial Position	4	3	2	1							
Move One	4	3	2		1							
Move Two	4	3			1				2			
Move Three	4	3							2	1		
Move Four												
Move Five												
Move Six												
Move Seven												
Move Eight												
Move Nine												
Move Ten												
Move Eleven												
Move Twelve												
Move Thirteen												
Move Fourteen												
Move Fifteen									4	3	2	1

[5]

- d) If the game is now played with n discs, conjecture a formula, in terms of n , for the smallest number of steps necessary to complete the puzzle, giving a reason for your answer. [1]

[Total for Question: 10]

END OF PAPER

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Paper Notes: 13+ Maths Question Paper (13+ Maths Past Paper (2021))

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Overview

This is **Mathematics A**, one of the examination papers set by **Eton College** for its **King's Scholarship Examination in 2021**. The King's Scholarship is awarded to the most academically able students entering the College at 13+, and this paper forms part of the rigorous assessment used to identify candidates worthy of that distinction. The paper is designed for **Year 8 pupils** applying for entry into Year 9, and it tests a broad range of mathematical topics at a level that extends well beyond the standard Key Stage 3 curriculum.

The paper is divided into two sections: **Section I** comprises short-answer questions worth a total of **50 marks**, whilst **Section II** consists of extended problem-solving questions also worth **50 marks**. Candidates have **one and a half hours** to complete the paper, and **calculators and protractors are not permitted**. This constraint places a premium on mental arithmetic, algebraic manipulation, and geometric reasoning without technological aids.

The paper is pitched at a high level and assumes familiarity with topics such as **fractional arithmetic, algebraic simplification, inequalities, geometry with surds, and combinatorial logic**. It is an excellent resource for students preparing for competitive 13+ entrance examinations at independent schools, or for those seeking challenging extension material in mathematics.

How this paper is organised

The paper opens with a cover sheet that provides clear rubrics: candidates must write their candidate number on every sheet, answer on the paper in the spaces provided, and are reminded that **no diagram is drawn to scale**. The prohibition on calculators and protractors is stated prominently.

Section I contains **ten short-answer questions** numbered 1 to 10, collectively worth **50 marks**. Individual questions range from **3 to 13 marks**, with marks for each part indicated in square brackets. The questions span arithmetic with mixed fractions, decimal calculations, algebraic substitution, simplification of nested brackets, solving inequalities, word problems involving ages and averages, angle geometry with parallel lines, and fractional equations.

Section II comprises **five extended questions** numbered 11 to 15, each worth **10 marks** and divided into multiple parts. These questions require more sustained reasoning and include problems on rates of work, cylinder geometry and surface area, conversion of recurring decimals to fractions, advanced Euclidean geometry involving surds and proofs, and a combinatorial investigation of the **Towers of Hanoi** puzzle. The rubric instructs candidates to answer all of Section I and as many questions as they can from Section II, suggesting that full completion of Section II may be challenging within the time limit.

Topics covered

- Arithmetic with mixed fractions: addition, subtraction, multiplication, and division of mixed numbers, with answers required as reduced mixed fractions
- Decimal arithmetic: multiplication and division of decimals, including calculations with negative numbers and cubes of decimals
- Algebraic substitution and simplification: evaluating expressions given specific values, simplifying nested brackets, and manipulating fractions involving algebraic terms
- Solving linear inequalities: rearranging inequalities to isolate the variable and expressing the solution as a mixed fraction
- Word problems involving rates: combined work problems (builders completing a task together), painting area problems with algebraic side lengths, and unit conversions between feet, miles, and hours
- Geometry of parallel lines and angles: calculating unknown angles using properties of alternate and corresponding angles formed by transversals cutting parallel lines
- Weighted averages: calculating the overall class average from separate averages for boys and girls, with attention to rounding to one decimal place
- Cylinder geometry: calculating ratios of volumes, deriving formulae for curved surface area, and finding the total exposed surface area of compound solids formed by joined cylinders
- Recurring decimals: converting recurring decimals to fractions by algebraic methods, including proofs that certain recurring decimals equal whole numbers
- Advanced Euclidean geometry with surds: working with semicircles, right-angled triangles, and midpoints; proving properties of squares; calculating lengths involving square roots; and using geometric constructions to derive inequalities involving π

How to use this paper for revision

- Practise manipulating mixed fractions without a calculator, focusing on finding common denominators and reducing answers to their simplest form before writing them down.
- Revise the rules for multiplying and dividing decimals by hand, and ensure you can handle negative bases raised to odd and even powers confidently.
- Work through algebra problems involving nested brackets slowly and methodically, removing one layer of brackets at a time and checking signs carefully at each step.
- When solving word problems, define your variables clearly at the start and write down the equations you are forming before attempting to solve them.
- For geometry questions, redraw diagrams where helpful and mark on all known angles and lengths, using properties of parallel lines, angles in semicircles, and Pythagoras where appropriate.
- Familiarise yourself with the standard technique for converting recurring decimals to fractions: multiply by an appropriate power of ten, subtract, and solve for the variable.
- In extended proof questions, write out each step of your reasoning in full sentences, justifying every statement with a geometric property or algebraic manipulation.

Common mistakes to avoid

- Failing to reduce fractions to their simplest form or leaving improper fractions instead of converting to mixed numbers, which will lose marks even if the numerical value is correct.
- Mishandling negative signs when substituting negative values into algebraic expressions, particularly when squaring or cubing negative numbers.
- Losing track of which brackets to expand first in nested algebraic expressions, leading to sign errors and incorrect collection of like terms.
- Forgetting to reverse the inequality sign when dividing or multiplying both sides of an inequality by a negative number.
- Confusing diameter and radius in cylinder problems, leading to errors by a factor of two or four in area and volume calculations.
- Rushing the Towers of Hanoi table in question 15 and making illegal moves (such as placing a larger disc on top of a smaller one), which invalidates the entire solution.

Exam technique

Begin with **Section I** and work through the questions in order, as they generally increase in difficulty. Allocate roughly **45 minutes** to Section I and **45 minutes** to Section II, leaving time at the end to check your work. If a question in Section I is taking too long, move on and return to it later rather than sacrificing time you could spend on the extended questions.

In **Section II**, read each question carefully and identify which parts you can attempt confidently. Even if you cannot complete every part of an extended question, partial credit is available for intermediate steps, so show all your working clearly. For proof questions, write out your reasoning in complete sentences and justify each step with a named theorem or property.

Use the space provided on the paper efficiently, but if you run out of room, continue on a blank page and indicate clearly where your answer is continued. Check that your final answers match the form requested in the question (mixed fractions, decimals to a specified number of places, expressions in terms of given variables). In the final few minutes, scan through your answers for arithmetic slips and ensure you have answered every part of every question you attempted.

What to revise alongside this paper

To prepare for a paper of this level, ensure you are confident with all Key Stage 3 algebra topics, including **expanding brackets**, **factorising expressions**, and **solving linear equations**. Revisit the rules for **order of operations** and the laws of indices, as these underpin much of the algebraic manipulation required.

For geometry, revise **properties of triangles** (including isosceles and right-angled triangles), **circle theorems** (particularly the angle in a semicircle), and **properties of quadrilaterals**. Practise problems involving **Pythagoras' theorem** and manipulation of expressions containing surds, as these appear in the more advanced geometry questions.

Beyond this paper, students aiming for King's Scholarships should explore **GCSE Higher Tier content** (algebraic fractions, solving quadratic equations, trigonometry) and consider extension resources such as **UKMT Intermediate Mathematical Challenge** past papers or **Olympiad-style problem books** to develop the problem-solving stamina required for Section II.

Key terms

Mixed fraction, Improper fraction, Recurring decimal, Linear inequality, Weighted average, Parallel lines, Transversal, Alternate angles, Corresponding angles, Cylinder, Curved surface area, Volume of a cylinder, Surd, Pythagorean theorem, Semicircle, Right angle in a semicircle, Midpoint, Proof by algebra, Towers of Hanoi, Combinatorial reasoning

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Eton College King's Scholarship Examination 2021

MATHEMATICS B

(One and a half hours)

Candidate number: _____

Please write your candidate number on EVERY sheet.

Please answer on the paper in the spaces provided.

There are 8 questions: each one is worth 10 marks.
Calculators are allowed, but you should show all your working.

Do not turn over until told to do so.

1. (a) By considering $(x + 1)(x - 1) = x^2 - 1$, find two prime factors of 899.

(b) Multiply out and simplify fully $(x + 1)(x^2 - x + 1)$.

- (c) Using this result, prove that
- (i) 27001 is divisible by 31

- (ii) 3376 is divisible by 211

(iii) $2^{48} + 1$ is divisible by 65537

(iv) $5^{18} + 1$ is divisible by 13

2. (a) I am buying musical instruments. A ukulele and a xylophone together cost £120 more than a violin. A violin and a xylophone together cost £180 more than a ukulele. How much does a xylophone cost?

- (b) Some of the instruments are presents for my godchildren. Because I dislike their parents, I have decided to buy each one either an accordion or a set of bagpipes. Accordions cost £80 and bagpipes cost £130. Two thirds of the presents will need to be sent by post: postage costs £30 per instrument. I spend £800 altogether. How many godchildren do I have? [Your working should demonstrate that your solution is unique.]

[space for continuation of solution to 2(b), if required]

3. (a) Show carefully by multiplying out and simplifying that $(4 + 3\sqrt{5})^2 = 61 + 24\sqrt{5}$.

(b) Multiply out and simplify $(\sqrt{7} - 2)^2$.

(c) Use the pattern you have observed to find an expression for the following in the form $a + b\sqrt{c}$, where a , b and c are integers.

(i) $\sqrt{7 + 2\sqrt{6}}$

(ii) $\sqrt{9 - 4\sqrt{5}}$

(iii) $\sqrt{3 - 2\sqrt{2}}$

(d) Find an expression for $\sqrt{4 + \sqrt{7}}$ in the form $\frac{1+\sqrt{q}}{\sqrt{p}}$, where p and q are integers.

4. Oysters are sold in pails by two rival companies, *Lion* and *Unicorn*. The Walrus and the Carpenter always divide up the contents of a *Lion* brand pail between themselves in the ratio 2:3, while they divide up a *Unicorn* brand pail in the ratio 5:4. The ratio of the total number of oysters in a *Lion* brand pail to that in a *Unicorn* brand pail is 3:5. If they cannot resist polishing off all the oysters in any pail they start, how many pails of each brand do they eat given that they consume the smallest total number of pails they need in order to ensure that they eat the same number of oysters each?

[space for continuation of solution to 4, if required]

5. Oggish is the language of the land of Og. Its alphabet consists of just two letters, 'o' and 'g'. Words may be formed by any combination of letters, but any word which contains the sequence of letters 'oo' is considered obscene and is not used in polite Oggish society. Thus, for example, the words 'oggg', 'gogogogo' and 'ggggggg' are considered polite, while 'oogo', 'ggggoooo' and 'googoog' are obscene.

(a) How many words in total (i.e. both polite and obscene) are there which contain

(i) exactly 3 letters?

(ii) exactly 4 letters?

(iii) exactly n letters?

(b) How many polite words are there which have

(i) exactly one letter?

(ii) exactly two letters?

(iii) exactly three letters?

(c) Oggo the Grammarian claims that if O_n is the number of polite Oggish words of n letters, then

$$O_n = O_{n-1} + O_{n-2}$$

(i) Use this formula to find the number of polite Oggish words which have exactly seven letters.

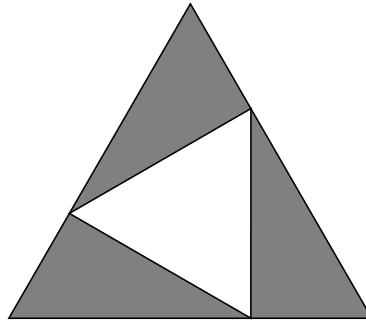
(ii) Explain Oggo's reasoning carefully.

6. (a) Tweedle-Dum and Tweedle-Dee decide to have a running race on a circular track. They start off in opposite directions from the finish line and stop when first they both meet there again. If Tweedle-Dum runs at a steady 7mph and Tweedle-Dee at a steady 8mph, how many times will they pass each other in between?

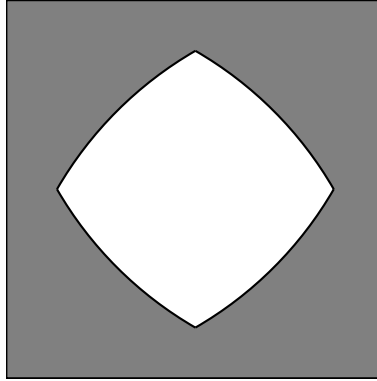
- (b) The Gryphon flies against the wind from the Duchess's House to the Queen's Croquet Ground in an hour and then flies back again with the wind in 80% of the time it would have taken him in still air. How long did the entire round trip take?

7. (a) Use Pythagoras's theorem to show that an equilateral triangle of perimeter 18 has an area of $9\sqrt{3}$.

- (b) The design for a stained glass window is shown below. The whole window is in the shape of an equilateral triangle. Three identical right-angled triangles (of coloured glass) just touch inside it, and enclose a triangle of plain glass. What is the ratio of coloured to plain glass in the whole window?

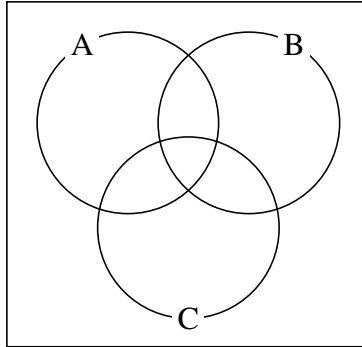


- (c) Another window has a design which is shown below. The whole window is a square; the central section is made of plain glass; the outer section is coloured. The curved lines are arcs of circles of the same radius as the side length of the square and centred on its corners. What is the ratio of coloured to plain glass in the whole window? Give your answer in the form $1 : x$, where x is a decimal correct to 3 sf.

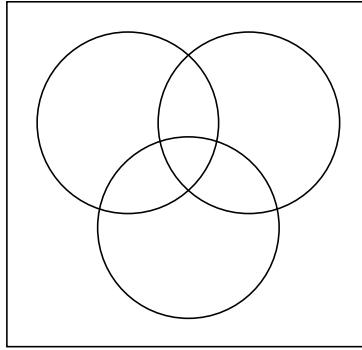


[space for continuation of solution to 7(c), if required]

8. (a) 57 people speak at least one of Aragonese, Basque and Castilian. 29 speak Aragonese; 34 speak Basque; 33 speak Castilian; 15 speak both Aragonese and Basque; 16 speak both Basque and Castilian; 12 speak both Aragonese and Castilian. How many speak all three languages? You may find it helpful to use the diagram in your answer.



- (b) Of the positive integers strictly less than 100, how many are divisible by at least one of 3, 5 and 7? You may find it helpful to use the diagram in your answer.



- (c) Groucho, Chico, Harpo, Gummo, Zeppo, Karl and Spencer must each choose to enrol in exactly one of three classes: Acrobatics, Ballet or Capoeira. In how many ways can this happen if there must be at least one student in each class? [Note that the actual participants in each class matter, not just the numbers in each class, i.e. “Groucho, Chico, Harpo, Gummo and Zeppo all choose Acrobatics, while Karl chooses Ballet and Spencer chooses Capoeira” is a different result from “Chico, Harpo, Gummo, Zeppo and Spencer all choose Acrobatics, while Karl chooses Ballet and Groucho chooses Capoeira”.]

END OF PAPER

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Paper Notes: 13+ Maths Question Paper (13+ Maths Past Paper (2021))

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Overview

This is the **Mathematics B** paper from **Eton College's King's Scholarship Examination 2021**, designed for candidates applying for a King's Scholarship at 13+ entry. The paper assesses advanced problem-solving and mathematical reasoning over **one and a half hours**, demanding depth of understanding across algebra, number theory, combinatorics, and geometry. Calculators are permitted, and candidates must show full working to earn credit.

The paper is structured as **eight free-response questions**, each worth **10 marks**, covering topics that extend well beyond the standard Key Stage 3 curriculum. Questions blend algebraic manipulation, proof, applied ratio problems, surd simplification, combinatorial counting, and geometric reasoning. Many problems are multi-part and build progressively, testing whether students can apply a single insight to related scenarios of increasing complexity.

This paper is aimed at exceptionally able 13-year-old candidates competing for prestigious academic scholarships. It is significantly harder than typical entrance or grammar-school 11+ papers, featuring questions that would challenge strong GCSE or even early A-level students. The inclusion of formal proof, recursive sequences, and inclusion-exclusion principles reflects the rigorous mathematical standard expected of King's Scholars at Eton.

How this paper is organised

The paper comprises **eight questions**, each allocated **10 marks**, for a total of **80 marks** across **90 minutes**. Questions are numbered 1 to 8, with many subdivided into parts (a), (b), (c), and occasionally (i), (ii), (iii), (iv). Each question occupies several pages, allowing generous space for extended working and explanations.

Questions progress in difficulty within each number: part (a) often establishes a technique or result, and subsequent parts require candidates to generalise or apply that insight. For example, Question 1 moves from factorising 899 using difference of squares, through expanding a sum-of-cubes identity, to proving divisibility for large powers. Similarly, Question 3 begins with squaring a binomial surd, then asks candidates to reverse-engineer square roots of nested surds.

The paper does not separate into distinct sections by topic. Instead, questions test a mixture of algebra, number theory, ratio and proportion, combinatorics, and geometry. Candidates should allocate roughly 11 minutes per question, though harder multi-part questions (such as Question 5 on the Oggish language or Question 8 on inclusion-exclusion) may require more time for careful reasoning.

Topics covered

- Algebraic factorisation and expansion, including difference of squares and sum of cubes identities
- Divisibility proofs using algebraic identities and substitution of specific integer values
- Simultaneous equations and systems of linear constraints with multiple variables
- Advanced ratio problems involving three-way comparisons and optimisation of total quantities
- Manipulation and simplification of surds, including squaring binomial surd expressions
- Nested surds and reverse-engineering expressions of the form $\sqrt{a + b\sqrt{c}}$ into $a + b\sqrt{c}$ form
- Complex word problems involving ratios, percentages, and constraints across multiple items or categories
- Combinatorial counting with restrictions, including recursion (Fibonacci-style sequences) and enumeration of valid words under lexical rules
- Circular motion and relative speed problems on a closed track
- Wind-effect problems involving average speed over a round trip
- Geometric reasoning with equilateral triangles, right-angled triangles, squares, and circular arcs
- Area calculations involving Pythagoras's theorem, nested shapes, and sector geometry
- Venn diagram reasoning and inclusion-exclusion principle for three overlapping sets
- Surjective counting problems (distributing distinguishable items into distinct categories with constraints)

How to use this paper for revision

- Practise algebraic proof techniques, especially factorising expressions and substituting specific values to demonstrate divisibility. Question 1 rewards fluency with sum-of-cubes and difference-of-squares identities.
- Revise the inclusion-exclusion principle for three sets thoroughly. Question 8 requires careful Venn diagram reasoning, and errors in overlapping regions cascade through the solution.
- Strengthen surd manipulation skills: square binomial surds, simplify nested radicals, and recognise when $\sqrt{a \pm b\sqrt{c}}$ can be rewritten as $p \pm q\sqrt{r}$. Question 3 tests this skill progressively.
- Master simultaneous equations with three or more variables. Question 2 and Question 4 require translating word problems into systems and solving for unknowns or integer constraints.
- Work on combinatorial recursion and pattern-spotting. Question 5 introduces a Fibonacci-like recurrence for polite Oggish words, and part (c)(ii) asks you to explain why the recursion holds.
- Revise geometric problem-solving involving overlapping shapes and sector areas. Question 7 combines Pythagoras, equilateral triangle properties, and circular arcs in a single multi-step problem.
- Practise showing full algebraic working even when a calculator is allowed. The paper awards marks for method, not just final answers, so every step must be clearly justified.

Common mistakes to avoid

- Forgetting to check that factors are prime when asked for prime factors (Question 1a). Candidates often stop at $899 = 29 \times 31$ without verifying both are prime.
- Losing track of which pails are Lion and which are Unicorn in Question 4. Mixing up the ratios (2:3 vs 5:4) or the pail-size ratio (3:5) leads to incorrect simultaneous equations.
- Miscounting valid Oggish words by failing to exclude words containing 'oo'. In Question 5, candidates must carefully distinguish polite from obscene words at every step.
- Incorrectly simplifying nested surds by treating $\sqrt{a + b\sqrt{c}}$ as $\sqrt{a} + \sqrt{b\sqrt{c}}$. Question 3(c) and (d) require recognising that $\sqrt{a + b\sqrt{c}} = (p + q\sqrt{r})$ and squaring both sides to find p , q , and r .
- Misapplying the inclusion-exclusion principle by double-counting or omitting the intersection of all three sets. Question 8 demands systematic Venn diagram bookkeeping.
- Rushing geometric area calculations and forgetting to subtract overlapping regions or account for sector angles. Question 7(c) requires careful identification of the coloured area as (square minus four quarter-circles).

Exam technique

Allocate roughly **11 minutes per question**, but be flexible. Skim the entire paper first to identify which questions suit your strengths, and attempt those early to build confidence and secure marks. Questions with multiple parts often allow partial credit, so complete easier sub-parts even if later sections seem daunting.

Show every step of algebraic working, even for straightforward expansions or factorisations. The mark scheme rewards method marks, and examiners need to see your reasoning to award credit if your final answer is wrong. Label parts clearly ((a), (b) (i), etc.) and box or underline final answers so they stand out. Use the generous space provided to lay out working logically, avoiding cramped or illegible notation.

For proof questions (Question 1(c), for example), state what you are proving at the start and conclude with a summary sentence ('Therefore 27001 is divisible by 31'). For combinatorial problems, draw diagrams or tables to organise cases systematically. In geometry questions, sketch diagrams and label sides and angles to clarify your reasoning. If you get stuck on a multi-part question, move on and return later; later parts sometimes offer hints about earlier ones.

What to revise alongside this paper

Strengthen your command of **GCSE-level algebra**, particularly expanding and factorising quadratics, cubics, and higher-degree polynomials. Revise algebraic proof by exhaustion and by substitution, and practise manipulating algebraic fractions and surds fluently. Study **modular arithmetic** and divisibility rules to underpin the number-theory questions.

Explore **combinatorics beyond the syllabus**: learn about permutations, combinations, the pigeonhole principle, and recursive counting. Investigate the Fibonacci sequence and its recurrence relation, as Question 5 builds directly on this pattern. Study **set theory and Venn diagrams** with three or more sets, and master the inclusion-exclusion formula for overlapping categories.

Revise **coordinate geometry and circle theorems** to support geometric reasoning with sectors and overlapping arcs. Practise problem-solving from **UKMT Intermediate and Senior Mathematical Challenges** and past Olympiad papers to develop the lateral thinking and proof-writing skills this paper demands. Work through **MAT, STEP, or TMUA** past papers (with support) to experience the style of multi-step, proof-based questions expected at this level.

Key terms

Difference of squares, Sum of cubes, Prime factorisation, Divisibility, Simultaneous equations, Ratio and proportion, Surd, Nested surd, Combinatorics, Recursion (Fibonacci sequence), Inclusion-exclusion principle, Venn diagram, Pythagoras's theorem, Equilateral triangle, Sector area

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