Eton College King's Scholarship Examination 2021

MATHEMATICS A

(One and a half hours)

Candidate Number:………………………………………………………

<u>Please write your candidate number on EVERY sheet</u>.

Please answer on the paper in the spaces provided.

This paper is divided into two sections:

Section I (Short-answer questions) – 50 marks available
Section II (Extended questions) – 50 marks available

Answer all of Section I and as many questions as you can from Section II.

The marks for each part of each question are given in square brackets.

Show all your working.

No diagram is drawn to scale.

Neither calculators nor protractors may be used.

<u>ADDITIONAL MATERIALS</u>: ***NONE***

Do not turn over until told to do so.

Section I: Short-answer questions (50 marks)

1. Find the value of the following, giving your answers as **reduced**, **mixed** fractions:

a) $\left(3\frac{2}{3}+\frac{2}{9}\right)\times 7\frac{2}{7}$ [3]

b) $\left(\frac{68}{19}\div\frac{17}{76}\right)\div\frac{6}{7}$ [4]

c) $327\frac{7}{12} + 271\frac{5}{9}$ [3]

d) $\left(4 - \frac{3}{4}\right)^2$ [3]

[Total for Question: 13]

2. Find the value of the following, giving your answer as a decimal:

a) 0.035×0.0022 [3]

b) $0.51 \div 0.068$ [3]

c) $(-1.1)^3$ [3]

[Total for Question: 9]

3. If $a = 1$ and $b = -2$, find the value of the following expressions, leaving your answers in simplified form:

a) $\frac{a}{b} - \frac{b}{a}$ [1]

b) $\frac{a^2+b^2}{a+b}$ [2]

[Total for Question: 3]

4. Simplify the following algebraic expressions fully, leaving no brackets in your final answers:

a) $2x - (3y + x) + \{3x - (5y - 4x + 7y)\}$ [2]

b) $a - [a - b - \{d - c + (a - b + c - d)\}]$ [2]

[Total for Question: 4]

5. Solve the following inequality, giving your final answer as a reduced, mixed fraction. In your final answer, x must appear on the left-hand side.

$$3 - 7x < 19 - 2x$$

[3]

[Total for Question: 3]

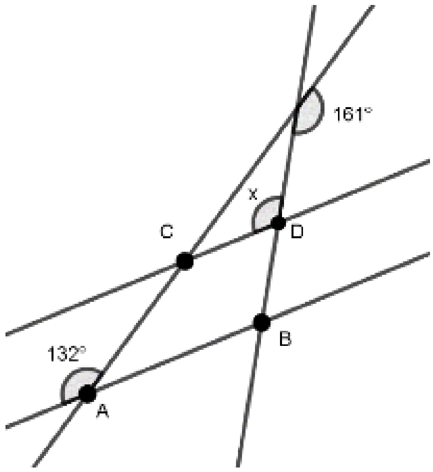
6. I have a glass containing five and two fifteenths fluid ounces of wine. I pour out one and seven twelfths fluid ounces of the wine. Find the volume of wine remaining in the glass, in fluid ounces as a reduced, **mixed** fraction: [4]

[Total for Question: 4]

7. Six years ago, Alice was 5 times as old as Beatrice was, but now she is only twice as old. Find the difference between the ages of Alice and Beatrice. [4]

[Total for Question: 4]

8. In the following diagram, line segments AB and CD are parallel. Calculate angle x.
[3]



[Total for Question: 3]

9. A class contains 30 pupils. 14 are boys and the rest are girls. In a test, the average mark of the boys is 62%, and the average mark of the girls is 68%. Find the average mark of the entire class, leaving your answer as a percentage correct to 1 decimal place.
[4]

[Total for Question: 4]

10. Solve the following equation, simplifying your final answer:

$$\frac{2}{3}\left(\frac{2x}{3}-3\right)-\frac{1}{6}\left(\frac{3x}{2}-8\right)=\frac{x}{12}$$

[3]

[Total for Question: 3]

Section II: Extended-answer questions (50 marks)

11. a) If one builder can build a wall in 5 hours, and a second builder can build one of the same size in 7 hours, how long will they take to build a wall working together? Give your answer in hours and minutes. [2]

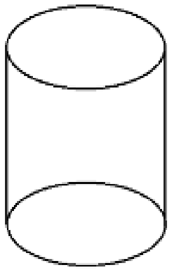
b) If one builder can build a wall in A hours, and a second builder can build one of the same size in B hours, how many hours will they take to build a wall working together? Leave your answer as a single fraction. [2]

c) If p litres of paint are required to paint a rectangular wall of side lengths $5q$ by q, how many litres of paint are required to paint a rectangular wall of side lengths $3r$ by r? [3]

d) A car travels at a rate of x feet in y seconds. How many hours does it take to travel z miles? You are given that 1 mile = 5280 feet. Leave your answer as a reduced, mixed fraction. [3]

[Total for Question: 10]

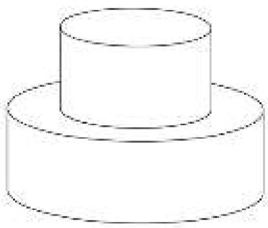
12. The volume of a cylinder is equal to the area of its circular base multiplied by its perpendicular height.



a) Suppose a cylinder has eight times the perpendicular height of a second cylinder and has a circular base one tenth the diameter of the second cylinder. What is the ratio of the volume of the two cylinders? [3]

b) Given that a suitably sized rectangular piece of paper can be wrapped around the curved surface of a cylinder so as to cover it exactly once with no overlap, write down a formula for the curved surface area of a cylinder in terms of the radius r of its base and its perpendicular height h. [2]

c) A child's toy is made of two solid cylinders joined together as illustrated. The larger cylinder has diameter $6p$ cm and perpendicular height $2p$ cm. The smaller cylinder has diameter $4p$ cm and perpendicular height $2p$ cm. Find a formula for the total exposed surface area of the toy, leaving your answer simplified and in terms of p and π. [5]



[Total for Question: 10]

13. Suppose that $x = 7.5\dot{3} = 7.53333\ ...$

a) i) Write down the value of $10x$ and $100x$. [2]

ii) Hence, prove that $90x = 678$ and deduce that $7.5\dot{3} = 7\frac{8}{15}$ [2]

Suppose now that $y = 1.\dot{9} = 1.9999\ ...$

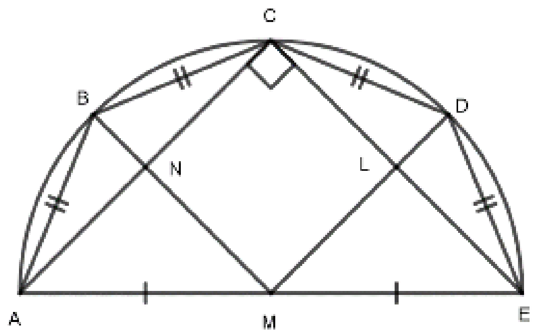
b) Using a method similar to part a), prove that $y = 2$. [2]

Suppose that $z = 17.b\dot{c} = 17.bcccc\ldots$, where the letters b and c represent digits between 0 and 9 occuring in the normal decimal representation of a number.

c) Find whole numbers u, v and w such that $z = \frac{u+vb+c}{w}$ [4]

[Total for Question: 10]

14. In the diagram, length AE equals $2 \times \sqrt{2}$ units and M is the mid-point of AE. Points B, C and D lie on a semicircle with diameter AE. Lengths AB, BC, CD and DE are all equal and angle ACE is a right angle.



a) Show that length AC equals 2 units. [1]

b) Prove that CLMN is a square. [2]

c) Show carefully that length DL equals $\sqrt{2} - 1$units. [2]

d) Show that length DE equals $\sqrt{4 - 2\sqrt{2}}$ units. [3]

e) Hence, prove that $\pi > 4 \times \sqrt{2 - \sqrt{2}}$. [2]

[Total for Question: 10]

15. *The Towers of Hanoi* is a puzzle consisting of three poles, labelled A, B and C, onto which punctured wooden discs of different sizes can be slid. Only one disc can be moved at a time, and at each stage every disc must be positioned so as to be smaller than the disc (if there is one) immediately beneath it. The object of the puzzle is to move the entire stack of discs, initially arranged vertically in order of decreasing size on pole A, to finish up arranged vertically in order of decreasing size on pole C.

a) For the first round, the game is played with two discs only. The smaller disc is labelled 1, and the larger disc 2. Complete the table below illustrating the shortest possible method of finishing the puzzle.

Stage	Pole A Lowest position → Highest		Pole B Lowest position → Highest		Pole C Lowest position → Highest	
Initial Position	2	1				
Move One	2		1			
Move Two						
Move Three					2	1

[1]

b) For the second round, the game is played with three discs, labelled 1, 2 and 3 in order of increasing size. Complete the table below illustrating the shortest possible method of finishing the puzzle.

Stage	Pole A Lowest position → Highest			Pole B Lowest position → Highest			Pole C Lowest position → Highest		
Initial Position	3	2	1						
Move One	3	2					1		
Move Two	3			2			1		
Move Three				2	1				
Move Four							3		
Move Five									
Move Six									
Move Seven							3	2	1

[3]

c) For the third round, the game is played with four discs. By completing the table below, show the smallest number of steps necessary to complete the puzzle is 15.

Stage	Pole A Lowest position → Highest				Pole B Lowest position → Highest				Pole C Lowest position → Highest			
Initial Position	4	3	2	1								
Move One	4	3	2		1							
Move Two	4	3			1				2			
Move Three	4	3							2	1		
Move Four												
Move Five												
Move Six												
Move Seven												
Move Eight												
Move Nine												
Move Ten												
Move Eleven												
Move Twelve												
Move Thirteen												
Move Fourteen												
Move Fifteen									4	3	2	1

[5]

d) If the game is now played with n discs, conjecture a formula, in terms of n, for the smallest number of steps necessary to complete the puzzle, giving a reason for your answer. [1]

[Total for Question: 10]

END OF PAPER