

# Eton College King's Scholarship Examination 2022

## MATHEMATICS B

(One and a half hours)

*Candidate number:*.....

**Please write your candidate number on EVERY sheet.**

**Please answer on the paper in the spaces provided.**

There are 8 questions: each one is worth 10 marks.

Calculators are allowed, but you should show all your working.

**Do not turn over until told to do so.**

1.

- (a) Find the acute angle between the hour hand and the minute hand of a (standard) 12-hour analogue clock at 4:30pm.
- (b) The point  $Q$  lies on the side  $AC$  of triangle  $ABC$  such that  $BQ$  and  $CQ$  are equal in length and the angle  $ABQ$  is half the angle  $BQC$ . Find the angle  $ABC$ .

- (c) The point  $R$  lies on the side  $DF$  of triangle  $DEF$  such that angle  $REF$  is 20 degrees greater than angle  $EFR$ , angle  $DER$  is a quarter of angle  $FRE$ , and angle  $RDE$  is one third of angle  $DRE$ . Find the angle  $RDE$  as a mixed fraction.

2.

- (a) A cuboid has two faces each with area 504 square inches, two faces each with area 924 square inches, and two faces each with area 66 square inches. Find the volume of the cuboid, showing your reasoning clearly.

- (b) A solid cuboid has four edges of length 10m, four edges of length 13m and four edges of length 15m. Vertices  $A$  and  $B$  are such that the diagonal  $AB$  is a space diagonal (that is, it passes through the interior of the cube). A thin piece of thread joins  $A$  and  $B$ , passing over the surface of exactly two of the faces of the cuboid and pulled tight to ensure that the thread is as short as can be. The length of thread between  $A$  and  $B$  is then measured.

Show that there are three possible lengths of thread, all lying between 27m and 30m, finding all three lengths to the nearest centimetre.

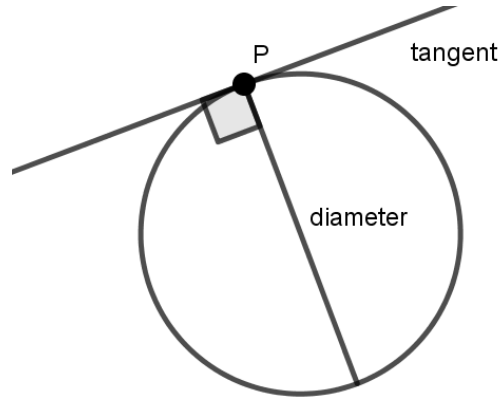
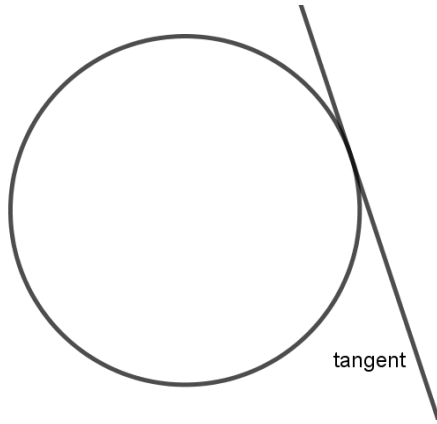
3.

- (a) Alice writes two lists of positive whole numbers. One contains all multiples of 144 less than 100,000,000. The other contains all multiples of 136 less than 100,000,000. How many numbers appear on exactly one of the lists?

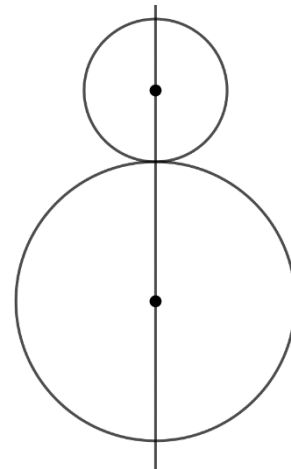
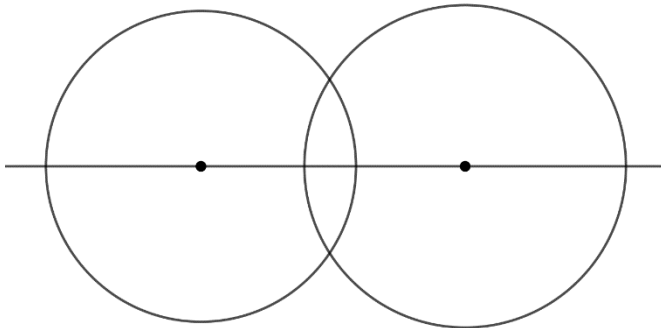
- (b) The Duchess is hosting a party and is sure that 760 people will attend. She knows that 532 of the guests like sandwiches, 608 like scones, 684 like cake and 722 like biscuits. Her cook is rather ill-tempered and so the Duchess is a little anxious about the catering. What is the smallest and the largest number of guests who could like all four foods?

4. *The following facts about circles will be useful for this question:*

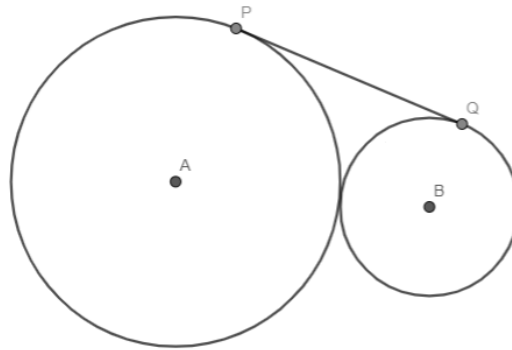
**Tangents and diameters:** A tangent to a circle is a straight line which touches the circle, at exactly one point, without crossing it. If a tangent touches a circle at the point  $P$ , the diameter to the circle from  $P$  is at 90 degrees to the tangent.



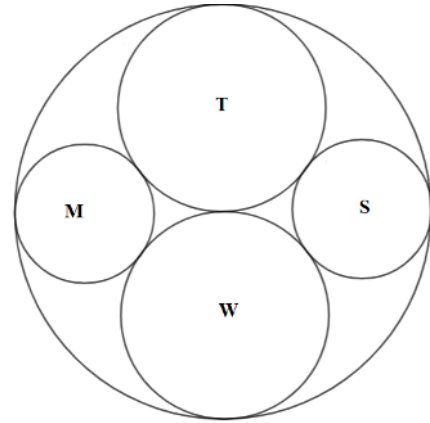
**Symmetry:** A straight line drawn through the centres of two circles is a line of symmetry for both circles.



- (a) The diagram shows two touching circles, one of diameter 72cm, centre  $A$  and one of diameter 50cm, centre  $B$ . The line segment  $PQ$  is a tangent to both circles. Find the length  $PQ$ .



- (b) The Mad Hatter has a new tea set. The design is shown from above. The teapot (T) and water-jug (W) are the same size, as are the (smaller) milk-jug (M) and sugar-bowl (S); all these items are of circular cross section, as is the tea-tray on which they sit. The items fit snugly on the tray, i.e. the circles just touch each other and the sides of the tray.



The teapot is of diameter 30cm.

What is the diameter of the milk-jug?

4.(b) cont.

5.

The Queen of Hearts keeps three pet flamingos, one in the drawing room, one in the library and one in the hallway. The flamingos squawk at fixed time intervals: in the drawing room every  $3\frac{1}{2}$  minutes, in the library every  $3\frac{1}{4}$  minutes and in the hallway every  $3\frac{1}{7}$  minutes.

All three squawk simultaneously at 12:00 noon on Monday, and next squawk simultaneously at time  $T$ .

- (a) Find the time  $T$ , which is on the Tuesday, giving your answer using the 24-hour clock.

- (b) Find on how many occasions between 12:00 noon on Monday and the time  $T$  that the drawing room and hallway flamingos squawk simultaneously but not the library flamingo.

6.

- (a) If  $p$  caterpillars can eat  $q$  mushrooms in  $t$  minutes, how long will  $r$  caterpillars take to eat  $s$  mushrooms, eating at the same rate?

- (b) The Queen's gardeners have three cylindrical water butts marked S, D and C respectively. They have different heights and different cross-sections. Each has a hole in its bottom; all the holes leak at exactly the same constant rate (i.e. the volume of water disgorged in a given time is the same for each). The butts are all full at noon. At 2pm, the depth of water in all the butts is the same. At 5pm, S runs dry. At 8pm, D runs dry. At 9:30pm, C runs dry. If D is 5cm taller than C, how tall is S?

6.(b) cont.

7.

- (a) In the Queen's garden, there are  $w$  white roses and  $r$  red roses. Each white rose has  $m$  petals and each red rose has  $n$  petals. Half of the white roses lose two thirds of their petals and one third of the red roses lose one third of their petals. What is the new average number of petals per rose?

- (b) The gardeners are diluting paint in order to paint the white roses red. They start with 10 litres of pure red paint in a paint kettle, but it is too thick, so they pour off a litre of paint into a bucket and pour a litre of water into the kettle before mixing thoroughly. The result is still too thick, so they pour off two litres of it and pour in two litres of water and then mix thoroughly. The result is still too thick, so they pour off three litres of it and pour in three litres of water and then mix thoroughly. Without checking the thickness of the resulting mixture, they pour off four litres of it and pour in four litres of water, only to discover on mixing that the final mixture is much too thin. They are resolved to do no more tampering, but they have kept the ten litres they have poured off in the bucket and discover to their relief that it is perfect for the job. What is perfect ratio of paint to water?

7.(b) cont.

8.

- (a) Points  $A, B, C, D, E$  and  $F$  all lie on the same straight line, in that order. The length of various line segments are known in terms of values  $x$  and  $y$ :

$$AD = 6x + y$$

$$CF = 8x - 4y$$

$$AF = 9x$$

$$BD = 8y - 4x$$

$$CE = 2x + y$$

Find the length of the line segment  $BE$  in terms of  $x$  and  $y$ .

- (b) The duck, the eaglet and the lory are fond of racing. Each always maintains a constant speed: the duck is the fastest and the lory the slowest of the three. Because of this, the eaglet always starts the race some distance ahead of the duck but some distance behind the lory.

In one race, the eaglet overtakes the lory, and five minutes later, the duck overtakes the lory and then three minutes after that, the duck overtakes the eaglet.

In another race, the duck overtakes the lory nine minutes after overtaking the eaglet. After how many more minutes will the eaglet overtake the lory?

8.(b) cont.

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