

MARKS SCHEME

Surname Candidate number

First name

Current school

TSA



The Manchester
Grammar School
Founded 1515

Entrance Examination 2024

Arithmetic Section B

1 Hour

Do not open this booklet until told to do so

Calculators may not be used

Write your names, school and candidate number in the spaces provided at the top of this page.

For each question, show all your working in full, as this will be marked, and then write your answer clearly in the space provided. If you run out of space for an answer use the space provided at the end of this booklet, numbering your answers carefully.

You have 1 hour for this paper which is worth 80 marks.

Marker	Short Problems Q1 - 6	Longer Problems Q7 - 11	TOTAL
Score	<input type="text"/>	<input type="text"/>	<input type="text"/>
out of	<input type="text" value="30"/>	<input type="text" value="50"/>	<input type="text" value="80"/>

1. You are given that $23 \times 372 = 8556$

Use the above information to find:

(a) 230×3.72

1a 855.6 ①

(b) $8556 \div 230$

1b 37.2 ①

(c) 2.3×3.72

1c 8.556 ①

(d) 23×18.6

1d 427.8 ①

(e) $4278 \div 186$

1e 23 ①

[5 marks]

2. A teacher wrote the following on the board, but left two spaces blank.

(a) Fill in the two blank spaces.

$1^3 = 1^2$	$1 = 1$
$1^3 + 2^3 = 3^2$	$3 = 1 + 2$
$1^3 + 2^3 + 3^3 = 6^2$	$\boxed{6} = \textcircled{1} + 2 + 3$
$1^3 + 2^3 + 3^3 + 4^3 = \boxed{10^2}$	$10 = 1 + 2 + 3 + 4$
$\textcircled{1}$	

(b) Now, in the boxes below, write out the whole of the next line of the pattern.

$1^3 + 2^3 + 3^3 + 4^3 + 5^3 = 15^2$

$15 = 1 + 2 + 3 + 4 + 5$

33 - 1 e.e.o.o.

[5 marks]

Please turn over

3. The planet Endor has an 18 hour "day", so the inhabitants of the planet, Endorians, use a nine hour clock to tell the time. They also use "am" and "pm" in exactly the same way that we do on Earth!

Remember to include "am" or "pm" where appropriate.

- (a) If it is 5pm on Endor, what time will it be in 7 hours time?

3a 3am

(1)

- (b) If it is 3am on Endor, what time was it 8 hours ago?

3b 4pm

(1)

- (c) How many full "days" would there be in 170 hours?

3c 9

(1)

- (d) On Endor, a particular task takes 8 hours. An Ewok performs this task 4 times taking just a single rest break. The Ewok noticed it was 1pm when they began, and it was also 1pm when they finished.

How long was the rest break?

3d 4 (hours) (1)

or 22
40
etc.

$$4 \times 8 = 32$$

(1)

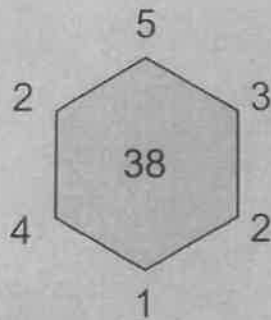
[5 marks]

Please turn over

4. The number written inside the hexagon is the sum of the **product** of the three numbers at the top and the **product** of the three numbers at the bottom.

(All numbers in this question are **positive whole numbers**)

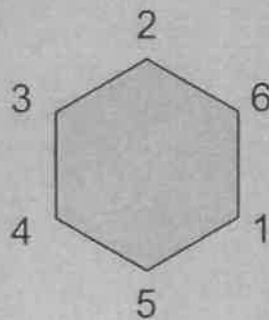
Example:



The number inside the hexagon is 38 because;

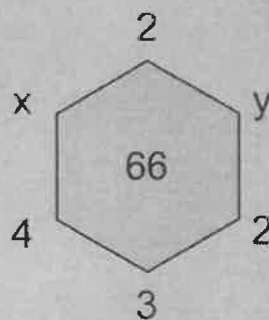
$$2 \times 5 \times 3 + 4 \times 1 \times 2 = 30 + 8 = 38$$

- a) Fill in the number that should go inside this hexagon.



4a 56 (1)

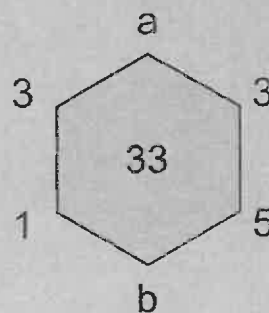
- b) Find a pair of numbers that **x** and **y** could stand for.



4b $x = 3$
 $y = 7$ (1)
(1)

or $x = 1$
 $y = 21$

- c) Find a pair of numbers that **a** and **b** could stand for.

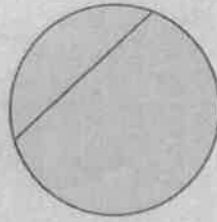


4c $a = 2$
 $b = 3$ (1)
(1)

[5 marks]

Please turn over

5. A "chord" of a circle is a straight line joining two points on the circumference (or edge) of a circle as shown in the diagram. This chord divides the circle into two regions.



If two different chords are drawn in a circle what is

- a) The **smallest** number of possible regions?

5a 3 (1)

- b) The **largest** number of possible regions?

5b 4 (1)

If three chords are drawn in a circle what is

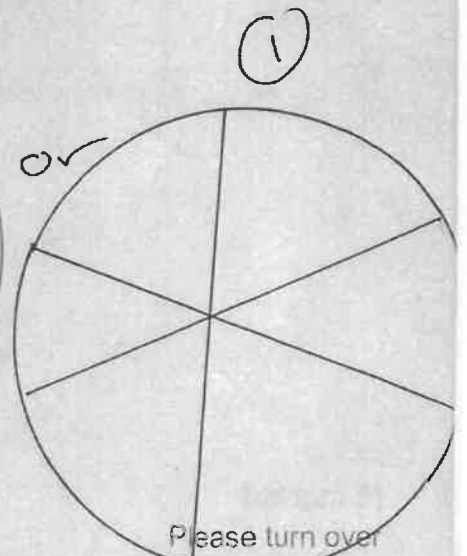
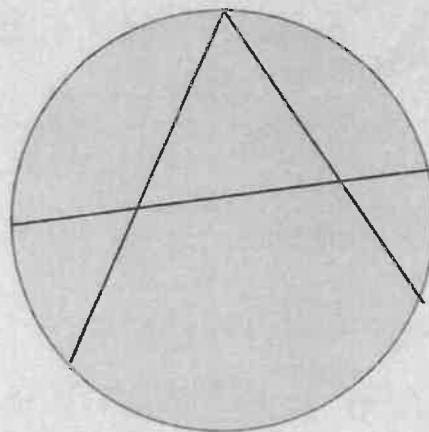
- c) The **smallest** number of possible regions?

5c 4 (1)

- d) The **largest** number of possible regions?

5d 7 (1)

In the circle below, draw **three** chords so as to create **six** regions.



[5 marks]

Please turn over

6. The "exp" of a two digit number is as follows:

The exp of 53 is $5 \times 5 \times 5 = 125$

The exp of 26 is $2 \times 2 \times 2 \times 2 \times 2 \times 2 = 64$

a) Work out the exp of 34

6a 81

①

b) What two digit number has an exp of 343?

6b 73

①

c) Find another two digit number that has the same exp as the exp of 28

6c 44

①

d) David finds the exp of a two digit number and then finds the exp of that answer.
If the final result is 9, what was the original number?

6d 25

①

$$9 \rightarrow 3^2 \rightarrow 32 \rightarrow 2^5 \rightarrow 25$$

Signat of 32, or 3^2 (m)

[5 marks]

FOR
MARKER
USE ONLY

Short problems	/30
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Please turn over

7. In Crazy maths there are only five digits, 1, 2, 3, 4 and 5.

Here are the addition and multiplication tables for Crazy maths:

+	1	2	3	4	5
1	2	3	4	5	1
2	3	4	5	1	2
3	4	5	1	2	3
4	5	1	2	3	4
5	1	2	3	4	5

x	1	2	3	4	5
1	1	2	3	4	5
2	2	4	1	3	5
3	3	1	4	2	5
4	4	3	2	1	5
5	5	5	5	5	5

Use the tables above to find:

a) $5 + 3$

7a (1)

b) 5×3

7b (1)

c) $(2 \times 3) \times 4$

7c (1)

d) $(3 \times 3) + (1 \times 4)$

7d (1)

e) If $m + m = 1$, what number is m standing for?

7e (1)

f) If $a + b = a \times b$ where a and b are **different** numbers, what numbers could a and b be standing for?

7f (1)
 either way

g) If $p \times q = q$ for all values of p , what is q standing for?

7g (1)

h) If $t \times t + t = 5$, what are two possible values of t ?

7h (1)

[10 marks]

Please turn over

8. This question only deals with **positive whole numbers**.

The **SPP** (Sum Plus Product) of two numbers is given by adding their sum and their product.

For example, the **SPP** of 2 and 7 is 23 because:

$$\text{SPP}(2, 7) = (2 + 7) + (2 \times 7) = 9 + 14 = 23$$

a) What is **SPP** (5, 9)?

8a 59 (1)

b) What is **SPP** (4, 9) - **SPP** (3, 2)?

8b 38 (1)

c) What number is x standing for if **SPP** ($x, 3$) = 31

8c $x =$ 7 (32)

$x + 3 + 3x = 31$
 $4x = 28$
 $x = 7$

M1 attempt
A1

← or answer w/out work scores 2

d) Find **all four** pairs of numbers (p, q) if **SPP** (p, q) = 11

8d 5, 1
1, 5
3, 2
2, 3

or 2 correct (31)
 all 4 correct (32)

2

e) What number is y standing for if **SPP** (y, y) = 48

8e $y =$ 6 (32)

$y^2 + 2y = 48$ any sensible attempt (M1)

← or answer at 6 without working scores 1

f) Find a and b , if b is three times the size of a and **SPP** (a, b) = 64

8f a = 4
b = 12 (32)

$\text{SPP}(a, 3a) = 4a + 3a^2 = 64$
 any sensible attempt (M1)

$a = 4$
 $b = 12$ (A1)

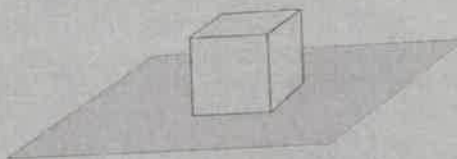
← or $a = 12$
 $b = 4$ scores 1

[10 marks]

Please turn over

9. The diagram shows a die (single dice) lying on a table. One of its faces is "hidden" but the other five faces can be seen.

The "total spot score" is the sum of the numbers on the faces that can be seen.
Remember that a normal die has the numbers 1 to 6 on it and opposite faces always add to 7.



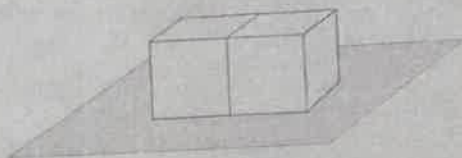
- a) What is the largest total spot score for this die.

9a 20 (1)

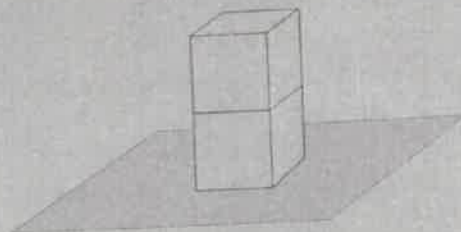
- b) What is the smallest total spot score for this die.

9b 15 (1)

When two dice are used, they can be placed in either position A or B



A



B

- c) In position A, how many of the dice faces are not hidden?

9c 8

- d) In position B, how many of the dice faces are not hidden?

9d 9

9. e) In position A, what is the largest **total spot score** possible?

9e 36 (1)

f) In position A, what is the smallest **total spot score** possible?

9f 20 (1)

g) In position B, what is the largest **total spot score** possible?

9g 34 (1)

h) In position B, if the **total spot score** is 33, what number is visible on the top face of the top die?

9h 5 (1)

if a third die is placed on top in position B, to make a tower 3 high,

i) what is the largest **total spot score** possible?

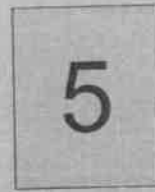
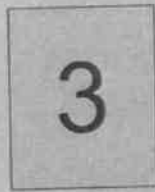
9i 48 (1)

j) what is the smallest **total spot score** possible?

9j 43 (1)

[10 marks]

10. Liam has some cards in a bag, and each card has a number on both sides. Liam takes two of the cards out of the box and they look like this:



The card with the 3 on it has a 7 on its other side, and the card with the 5 on it has a 2 on its other side.

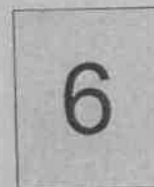
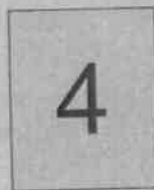
Liam throws the cards in the air and adds the two numbers he sees, when they land.

- a) What are the possible **totals** Liam could get?

10a 5, 8, 9, 12

~~5, 8, 9, 12~~
~~5, 8, 9, 12~~
 2 correct (5, 9)
 or all 4 correct (5, 8, 9, 12)

Next, he takes **two other** cards out of the box and they look like this:



After throwing these two in the air he finds the totals he can make are 7, 8, 10 and 11.

- b) What number could be on the other side of the 4 card?

10b 1

or 5

- c) And **for your answer in b)** what then is the number on the other side of the 6 card?

10c 7

or 3

must be paired correctly
 ie 1 with 7
 or 5 with 3

Liam then takes a **further three cards** out of the box and they look like this:



After throwing them, he sees the possible totals are 14, 15, 16, 17, 19, 20, 21 and 22. He also sees that **in no particular order**, the numbers 1, 5 and 9 are on the other sides.

d) What is the number on the other side of the 8 card?

10d	9
-----	---

(32)

e) What is the number on the other side of the 7 card?

10e	5
-----	---

(32)

f) What is the number on the other side of the 6 card?

10f	1
-----	---

(32)

[10 marks]

11. Given a pair of numbers (a, b) the following three rules are defined as:

Rule A changes (a, b) to (b, a)

Rule B changes (a, b) to $(a + b, a - b)$

Rule C changes (a, b) to $(3b, 2a)$

Examples:

A $(5, 2)$ becomes $(2, 5)$

B $(5, 2)$ becomes $(7, 3)$

C $(5, 2)$ becomes $(6, 10)$

If more than one rule is used, then they are applied in order from right to left, so for example

AB $(5, 2)$ means:

apply rule B first to $(5, 2)$ giving $(7, 3)$ then apply rule A to $(7, 3)$ giving $(3, 7)$

a) Find B $(11, 7)$

11a	18, 4
-----	-------

b) Find BA $(3, 8)$

11b	11, 5
-----	-------

c) Find CAB $(9, 1)$

11c	30, 16
-----	--------

$$\begin{aligned}
 B(9, 1) &= (10, 8) \text{ — either } \textcircled{1} \\
 A(10, 8) &= (8, 10) \text{ — } \textcircled{1} \\
 C(8, 10) &= (30, 16) \text{ — } A \textcircled{1}
 \end{aligned}$$

or

d) Find x & y if $B(x, 2) = (y, 3)$

11d $x = 5$
 $y = 7$

$$B(x, 2) = (x+2, x-2) = (y, 3)$$

attempt at

(M1)
(A1)

or

(B2)

e) Find x & y if $AC(x, 5) = (6, y)$

11e $x = 3$
 $y = 15$

$$AC(x, 5) = A(15, 2x) = (2x, 15) = (6, y)$$

(M1) attempt at \uparrow
(A1)

or

(B2)

f) Find x if $ABC(x, 3) = (7, 11)$

11f |

(B2)

$$\begin{aligned} ABC(x, 3) &= AB(9, 2x) \\ &= A(9+2x, 9-2x) \\ &= (9-2x, 9+2x) = (7, 11) \end{aligned}$$

(M1) getting to at least here

(A1)

[10 marks]

This is the end of the Examination

Use any remaining time to check your work or try any questions you have not answered.

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Longer
problems

/50

(B2)

(1)

(1)