



# TONBRIDGE SCHOOL

Scholarship Examination Sample Paper

## MATHEMATICS II

**Time allowed: 1 hour 30 minutes**

*There are seven questions in this paper.  
Answer as many questions as you can.  
All the questions carry equal marks.  
You may attempt the questions in any order.*

*All answers must be supported by adequate explanation.  
Calculators may be used in any question.*

1. The following equation is used in the manufacture of the lenses of glasses:  $\frac{P}{n-1} = \frac{1}{Q} - \frac{1}{R}$ , where  $P$  is the power of the lens,  $n$  depends on what the lens is made of, and  $Q, R$  are related to the curvature of the two surfaces of the lens.

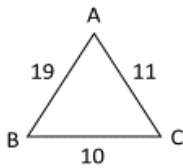
Note that any of  $P, Q, R$  may be negative numbers, but  $n$  is always positive.

In the questions below, give your answers as decimals correct to 2 decimal places.

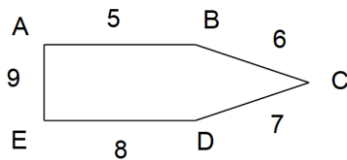
- (a) If  $n = 1.5, Q = 0.4, R = -0.7$ , find  $P$ .
- (b) If  $P = 2.6, n = 1.7, Q = 0.3$ , find  $R$ .
- (c) If  $P = 0.2, Q = 2.4, R = 3.7$ , find  $n$ .
- (d) If  $P = 1.8, n = 1.4$  and  $R = 2Q$ , find  $Q$ .

2. In the following two diagrams, the number on each edge is the sum of the two numbers at the ends of the edge. For example, in (a), we have  $A + B = 19$ .

- (a) By forming and solving appropriate equations, find the numbers  $A, B, C$ .



- (b) In a similar way, find the numbers  $A, B, C, D, E$ .

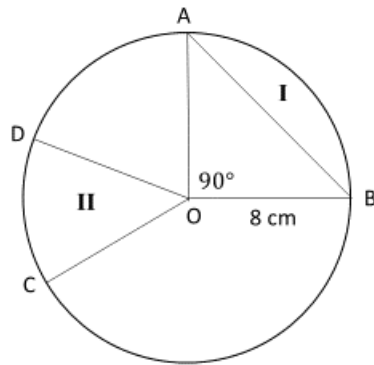


3. [In this question you will need to use the facts that a sphere of radius  $r$  has curved surface area  $S = 4\pi r^2$  and volume  $V = \frac{4}{3}\pi r^3$ .]

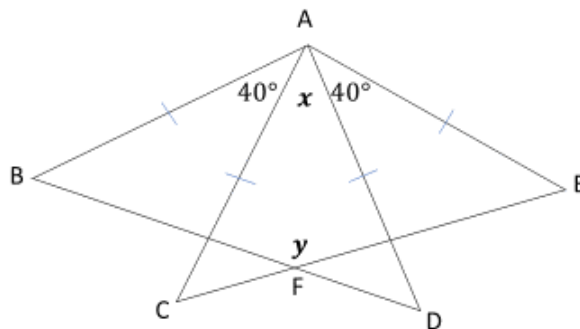
This question concerns hollow spheres which are the solid shapes formed by removing a small inner sphere of radius  $r$  cm from a large solid sphere of radius  $R$  cm.

- (a) If  $R = 9$  and  $r = 6$ , find the volume of the hollow sphere and its total (inner and outer) surface area.
- (b) If  $R = 12$  and the volume of the hollow sphere is  $5000 \text{ cm}^3$ , find  $r$ .
- (c) Find positive integer values of  $R$  and  $r$  for which the hollow sphere has the same total (inner and outer) surface area as that of a solid sphere of radius  $10$  cm.

4. The figure below shows a circle centre  $O$  with radius 8 cm and angle  $\angle AOB = 90^\circ$ .  
 Region I is the segment bounded by the straight line  $AB$  and the arc  $AB$ .  
 Region II is the sector bounded by the radii  $OC, OD$  and the arc  $CD$ .  
 Region I and Region II have equal areas.



- (a) Find the area of Region I.
- (b) By considering what fraction of the whole circle sector  $COD$  occupies, find angle  $\angle COD$ .
- (c) Which of Region I or Region II has the larger perimeter?  
 [You need to give calculations to justify your answer.]
5. In the diagram, the triangles  $ABC, ACD$  and  $ADE$  are isosceles with  $\angle BAC = \angle DAE = 40^\circ$ .  
 Also, angle  $\angle CAD = x$  and angle  $\angle BFE = y$ .



- (a) If  $x = 30^\circ$ , find  $y$ .
- (b) Use algebra to show that, whatever the value of  $x$ , you always get the same answer for  $y$  that you got in (a).

**TURN OVER**

6. This question concerns the graph whose equation is  $y = \frac{1}{x} + \frac{5x^2}{x^2+4}$ .
- Make a table of  $y$  values (correct to 2 decimal places) first for  $x$  values starting at  $x = 0.2$  and going to  $x = 1$  in steps of 0.2 and also for  $x = 2, 4, 6, 8, 10$ .
  - Why is  $x = 0$  not included in the table of values?
  - Using all of your values in (a) and choosing sensible scales, plot a graph of  $y$  against  $x$ .
  - Write down the coordinates of two points on the graph whose  $x$  and  $y$  coordinates are both positive integers with  $1 \leq x \leq 10$ . Explain carefully why there are no other such points on the graph.
7. Study the pattern of numbers in the table below carefully. Column B gives the answer to the sum in Column A; Column C gives the number in Column B in a factorised form.

	<b>A</b>	<b>B</b>	<b>C</b>
<b>Row 1</b>	$1 \times 4$	4	$1 \times 4$
<b>Row 2</b>	$1 \times 4 + 2 \times 7$	18	$2 \times 9$
<b>Row 3</b>	$1 \times 4 + 2 \times 7 + 3 \times 10$	48	$3 \times 16$
<b>Row 4</b>			
<b>Row 5</b>			
<b>Row <math>n</math></b>			

- Write down the entries in Columns A, B and C for Row 4 and Row 5.
- For Row  $n$ , find a formula in terms of  $n$  for the two numbers at the right-hand end of Column A and for the entry in Column C.
  - If the entry in Column B is 7600, what is the entry in Column C?
  - If the two numbers at the right-hand end of Column A multiply to give 444, what is the entry in Column B?
- Explain carefully why the entries for Column B in Row  $n$  have values that are between  $n^3$  and  $(n + 1)^3$ .
- Use (d) to help you find the value of  $n$  for which the entry in Column B is 5237748.

**END OF PAPER**