



TONBRIDGE SCHOOL

Scholarship Examination Sample Paper

MATHEMATICS II

Time allowed: 1 hour 30 minutes

*There are seven questions in this paper.
Answer as many questions as you can.
All the questions carry equal marks.
You may attempt the questions in any order.*

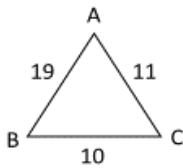
*All answers must be supported by adequate explanation.
Calculators may be used in any question.*

1. The following equation is used in the manufacture of the lenses of glasses: $\frac{P}{n-1} = \frac{1}{Q} - \frac{1}{R}$, where P is the power of the lens, n depends on what the lens is made of, and Q, R are related to the curvature of the two surfaces of the lens.
Note that any of P, Q, R may be negative numbers, but n is always positive.
In the questions below, give your answers as decimals correct to 2 decimal places.

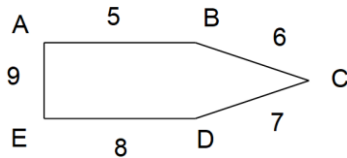
- (a) If $n = 1.5, Q = 0.4, R = -0.7$, find P .
- (b) If $P = 2.6, n = 1.7, Q = 0.3$, find R .
- (c) If $P = 0.2, Q = 2.4, R = 3.7$, find n .
- (d) If $P = 1.8, n = 1.4$ and $R = 2Q$, find Q .

2. In the following two diagrams, the number on each edge is the sum of the two numbers at the ends of the edge. For example, in (a), we have $A + B = 19$.

- (a) By forming and solving appropriate equations, find the numbers A, B, C .



- (b) In a similar way, find the numbers A, B, C, D, E .

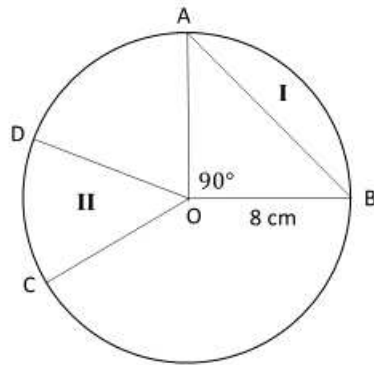


3. [In this question you will need to use the facts that a sphere of radius r has curved surface area $S = 4\pi r^2$ and volume $V = \frac{4}{3}\pi r^3$.]

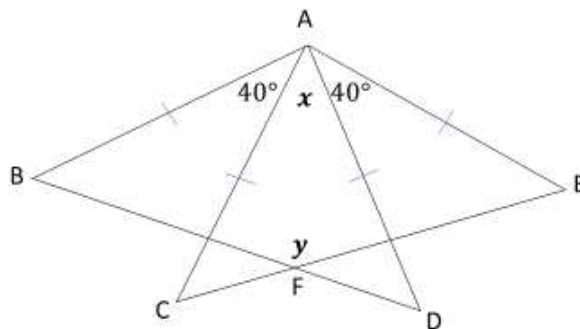
This question concerns hollow spheres which are the solid shapes formed by removing a small inner sphere of radius r cm from a large solid sphere of radius R cm.

- (a) If $R = 9$ and $r = 6$, find the volume of the hollow sphere and its total (inner and outer) surface area.
- (b) If $R = 12$ and the volume of the hollow sphere is 5000 cm^3 , find r .
- (c) Find positive integer values of R and r for which the hollow sphere has the same total (inner and outer) surface area as that of a solid sphere of radius 10 cm.

4. The figure below shows a circle centre O with radius 8 cm and angle $\angle AOB = 90^\circ$.
 Region I is the segment bounded by the straight line AB and the arc AB .
 Region II is the sector bounded by the radii OC, OD and the arc CD .
 Region I and Region II have equal areas.



- (a) Find the area of Region I.
- (b) By considering what fraction of the whole circle sector COD occupies, find angle $\angle COD$.
- (c) Which of Region I or Region II has the larger perimeter?
 [You need to give calculations to justify your answer.]
5. In the diagram, the triangles ABC, ACD and ADE are isosceles with $\angle BAC = \angle DAE = 40^\circ$.
 Also, angle $\angle CAD = x$ and angle $\angle BFE = y$.



- (a) If $x = 30^\circ$, find y .
- (b) Use algebra to show that, whatever the value of x , you always get the same answer for y that you got in (a).

TURN OVER

6. This question concerns the graph whose equation is $y = \frac{1}{x} + \frac{5x^2}{x^2+4}$.
- Make a table of y values (correct to 2 decimal places) first for x values starting at $x = 0.2$ and going to $x = 1$ in steps of 0.2 and also for $x = 2, 4, 6, 8, 10$.
 - Why is $x = 0$ not included in the table of values?
 - Using all of your values in (a) and choosing sensible scales, plot a graph of y against x .
 - Write down the coordinates of two points on the graph whose x and y coordinates are both positive integers with $1 \leq x \leq 10$. Explain carefully why there are no other such points on the graph.
7. Study the pattern of numbers in the table below carefully. Column B gives the answer to the sum in Column A; Column C gives the number in Column B in a factorised form.

	A	B	C
Row 1	1×4	4	1×4
Row 2	$1 \times 4 + 2 \times 7$	18	2×9
Row 3	$1 \times 4 + 2 \times 7 + 3 \times 10$	48	3×16
Row 4			
Row 5			
Row n			

- Write down the entries in Columns A, B and C for Row 4 and Row 5.
- For Row n , find a formula in terms of n for the two numbers at the right-hand end of Column A and for the entry in Column C.
- If the entry in Column B is 7600, what is the entry in Column C?
 - If the two numbers at the right-hand end of Column A multiply to give 444, what is the entry in Column B?
- Explain carefully why the entries for Column B in Row n have values that are between n^3 and $(n + 1)^3$.
- Use (d) to help you find the value of n for which the entry in Column B is 5237748.

END OF PAPER