

WESTMINSTER SCHOOL
THE CHALLENGE 2022

MATHEMATICS III

Wednesday 27 April 2022

Time allowed: 1 hour 30 minutes

You may not use a calculator for this paper.

All your working should be clearly shown.

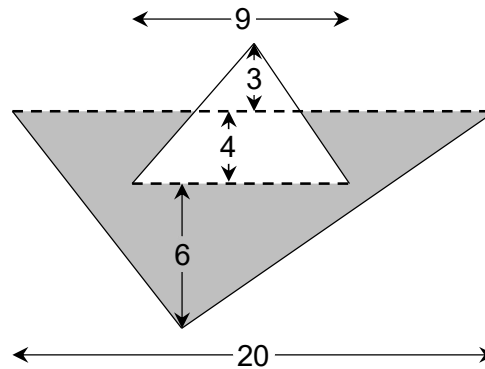
You should attempt all the questions.

Please write in black or blue ink.

- 1 Solve the following simultaneous equations.

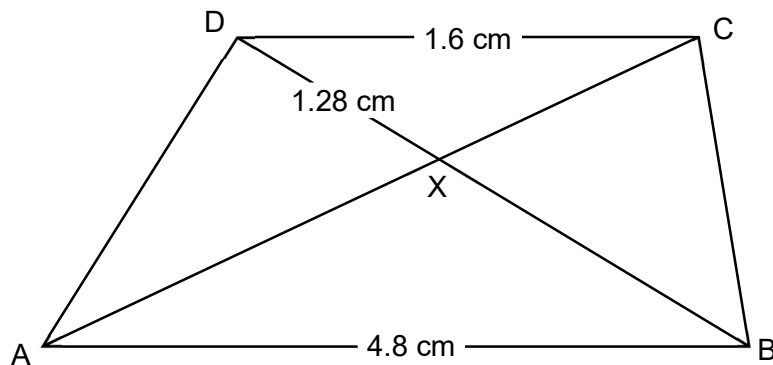
$$\begin{aligned}x + 2y + 3 &= 3(x + 5) \\ 3x - 2y + 1 &= -3(y + 1)\end{aligned}$$

- 2 It takes thirty sandsmiths to make 11,200 grains of sand in sixteen hours. How many sandsmiths does it take to make 14,700 grains of sand in eighteen hours?
- 3 The train from London Kings Cross station to Edinburgh Waverley station covers 420 miles and a ticket costs £189. The train from Leeds station to Plymouth station covers 350 miles and a ticket costs £154. Which train journey costs less per mile?
- 4 The diagram below shows two triangles. The dashed lines are parallel. All lengths shown are in cm. Find the shaded area.

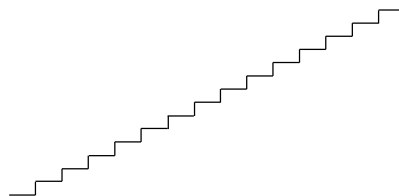


- 5 At a clown college, some students can juggle and some can ride a unicycle. Some students are able to do both, and some can do both simultaneously.
- Of the students who can juggle, two-fifths can ride a unicycle, but only three-tenths can juggle while riding a unicycle. Of the students who can ride a unicycle, one-sixth can juggle while riding a unicycle.
- What proportion of the students who can ride a unicycle are able to juggle?
- 6 Four stoats and ten weasels will eat a sack of food in 44 minutes. Eleven stoats and fifteen weasels will eat a sack of food in 24 minutes. How long will it take twenty stoats to eat a sack of food?
- 7 a Expand and simplify the expression $(3x - 1)^2$.
b Use this to find 2999^2 .

- 8 The diagram below shows a trapezium ABCD with AB parallel to CD. Diagonals AC and BD meet at the point X. DC = 1.6 cm, AB = 4.8 cm and DX = 1.28 cm.



- a By finding a pair of similar triangles, calculate the length BX.
- b You are given that triangle AXD has area k cm². Find an expression for the area of triangle ADB in terms of k , fully justifying your answer.
- 9 A twenty pence coin weighs 5 g. A fifty pence coin weighs 8 g.
- a A set of such coins has a total value of £9.10 and a total weight of 187 g. How many of each type of coin is there in the set?
- b What is the greatest possible value for a set of such coins weighing 187 g?
- 10 At the ink factory, they mix cyan and magenta ink to make new inks.
- Blue Steel* is made of cyan and magenta in the ratio 13 : 5.
Veronica Lake is made of cyan and magenta in the ratio 4 : 11.
Cerulean Sweater is made of cyan and magenta in the ratio 17 : 10.
- The factory has a surplus of Blue Steel and Veronica Lake and a shortage of Cerulean Sweater. In what ratio could they mix Blue Steel and Veronica Lake in order to make Cerulean Sweater?
- 11 Rocky and Apollo are running continuously up and down a flight of stairs. They run at different speeds, but each athlete runs 25% faster down the stairs than he runs up the stairs.

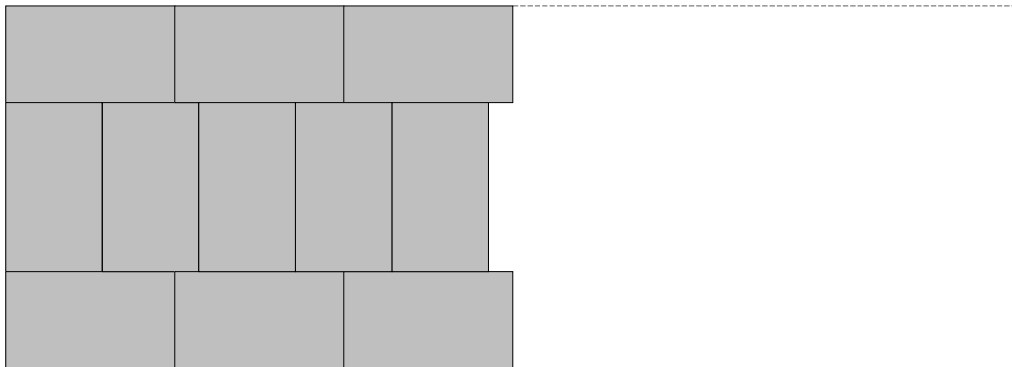


Rocky starts at the bottom of the stairs and Apollo starts at the top. The first time they pass each other, they are two-fifths of the way from the bottom of the stairs.

Where are they when they pass each other for the second time?

QUESTIONS CONTINUE OVERLEAF

- 12** I have a supply of 100 identical paving slabs of length 68 cm and width 41 cm, which I am using to create a path over a muddy field. The paved area will take the shape of a rectangle, and there must be no gaps. I use the following pattern:



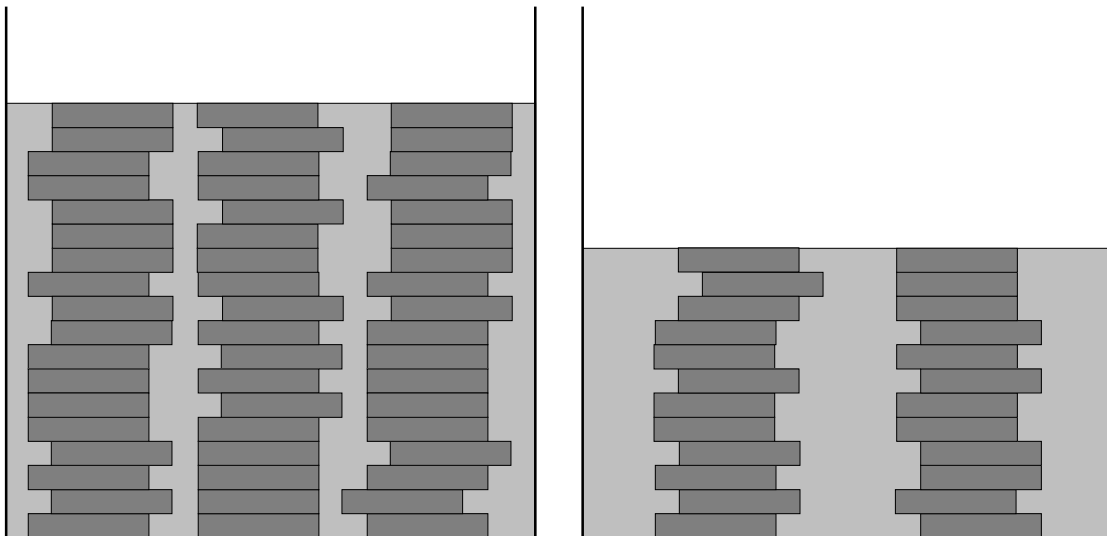
I lay the slabs lengthwise along the edges of the path and crosswise in the centre of the path, as shown in the diagram. When I have placed the final slabs in the pattern, I can, if necessary, make a single straight cut across the end of the path with my chainsaw, in order to leave an exactly rectangular paved area.

What is the longest paved area that I can make?

- 13** Pennies are thrown into a small well for good luck. The well is cylindrical, vertical and watertight, containing a fixed amount of water.

If the coins form three columns in the water, it takes 54 pennies in total for all columns to reach the surface of the water, as shown in the diagram on the left below.

If the coins form two columns in the water, it takes 24 pennies in total for both columns to reach the surface of the water, as shown in the diagram on the right below.



If the coins form a single column in the water, how many pennies would it take for this column to reach the surface of the water?

END OF QUESTIONS