



WESTMINSTER SCHOOL
THE CHALLENGE 2023

MATHEMATICS III

Wednesday 26 April 2023

Time allowed: 1 hour 30 minutes

You may **not** use a calculator for this paper.

All your working should be clearly shown.

You should attempt all the questions.

Please write in black or blue ink.

Write your answers in the answer booklet provided.

- 1 Solve the following simultaneous equations.

$$x + y = 4(y + 5)$$

$$y + 5 = 3(x - y)$$

- 2 It costs £10.85 to buy two plates, four bowls and one mug.
It costs £13.95 to buy four plates, three bowls and two mugs.
How much does it cost to buy two plates and one mug?

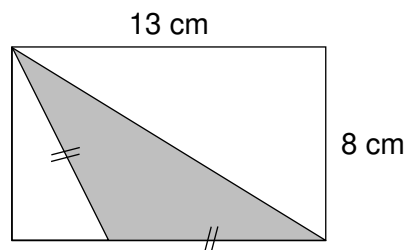
- 3 Seven shredders can destroy 93 600 sheets of paper in exactly three hours.
How long would it take five shredders to destroy 72 800 sheets of paper?

- 4 Find the value of the first term in the sequence below which exceeds 10 000.

239 373 507 641 775 ...

- 5 My electric scooter has two settings: 12 miles per hour and 17 miles per hour.
I want to get to the train station, three miles away by road, in exactly thirteen minutes.
If I start on the slow setting, after how much time should I switch to the fast setting?

- 6 Find the area of the shaded isosceles triangle shown in the rectangle below.
Give your answer as a mixed fraction in square centimetres.



- 7 Every basilisk has two eyes, and each eye is either yellow or green.

Among basilisks which have a yellow left eye, two-thirds have a yellow right eye.
Among basilisks which have a green left eye, three-fifths have a green right eye.
Among basilisks which have only one eye colour, three-quarters have green eyes.

What proportion of basilisks have two different eye colours?

- 8 **a** Expand and simplify the expression $(5x - 7)(5x + 7)$.
b Hence, or otherwise, find all of the prime factors of 2451.

- 9 A 'quarter' is an American coin worth \$0.25, and at the Retro Arcade there are only two machines which take quarters:

- *Pentis* provides 20 seconds of entertainment per quarter inserted.
- *Tac-Man* provides 57 seconds of entertainment per quarter inserted.

What is the maximum length of time that I could keep both of my children entertained simultaneously on these two machines, if I have \$10 in quarters?

- 10 Werewolves, hobgoblins, gremlins and unicorns sometimes come out at night.

All werewolves come out on the same night, every thirty nights.

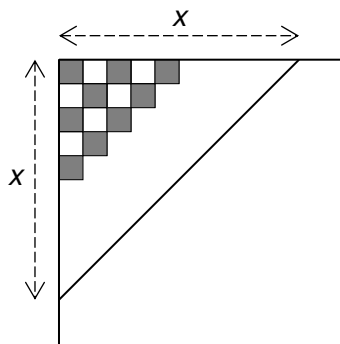
All hobgoblins come out on the same night, every eighteen nights.

All gremlins come out on the same night, every Friday night.

Unicorns only come out on nights when werewolves, hobgoblins, and gremlins all come out together. Unicorns were last seen on the night of 31 March 2023.

On which date will unicorns next be seen?

- 11 My kitchen floor is tiled with grey and white tiles of side length 1 foot, in an alternating pattern which continues as begun below. Find the length x such that the isosceles triangle shown in the corner of the room contains exactly 23 square feet of grey area.



- 12 *Lithobius forficatus* is a species of centipede with 30 legs.
Scolopendra gigantea is a species of centipede with 46 legs.

Find all possible combinations of such centipedes, if there are any, which would have:

- a 796 legs in total.
- b 804 legs in total.

QUESTIONS CONTINUE OVERLEAF

13 The 200 m Individual Medley is a swimming race held in a pool of length 50 m. Each swimmer must complete:

- one length of butterfly, then
- one length of backstroke, then
- one length of breaststroke, then
- one length of freestyle.

Wellgunde and Flosshilde are competing in this race. Each of these swimmers has fixed speeds at which she swims each of the four strokes above.

Both swimmers are as fast at butterfly as each other.
Each swimmer is as fast at freestyle as she is at backstroke.
Wellgunde swims breaststroke 25% faster than Flosshilde does.

The swimmers finish the race at the same time.
Where was Flosshilde when Wellgunde began her final length?

END OF QUESTIONS