NAME: _____

SCHOOL: _____

WINCHESTER
COLLEGE

WINCHESTER ELECTION

Mathematics II

Tuesday 30th April 2024

Time Allowed: 90 minutes

Total Marks: 100

Additional Information:

CALCULATORS ARE NOT ALLOWED.

Write your answers in this booklet. If you need additional space, please write on sheets of A4 paper and attach them to this booklet. You should show all your working so that credit may be given for partly correct answers.

Diagrams are not drawn to scale.

Do not be discouraged if you do not finish.

1. Evaluate:

a) $121 + 8 \times 121 + 2(121)$. [1]

b) $5 \times 2^3 \times 3 \times 2 \times 5^2$. [1]

c) 5% of 25% of 50% of 3200. [1]

d) $\frac{\sqrt[3]{64}^2}{\sqrt{4^3}}$. [2]

e) $\frac{\sqrt{1.44}}{\sqrt{0.04}}$. [2]

f) $\frac{(-27)^4}{9^5}$. [2]

g) Determine n, where $2^n = 2^3 + 2^3 + 2^3 + 2^3$. [2]

h) What fraction is halfway between $\frac{1}{8}$ and $\frac{1}{9}$. [2]

2. Solve:

a) $\dfrac{81-x}{2x+6}=3.$ [2]

b) $3(2x+5)-2(6-2x)=43.$ [2]

c) $\dfrac{5}{4-4x}=\dfrac{2}{x+12}.$ [2]

d) Solve $\dfrac{3x^3}{8}+25=1$. [2]

e) Expand and simplify
$2a(2b+3a)-3b(3b-2a).$ [2]

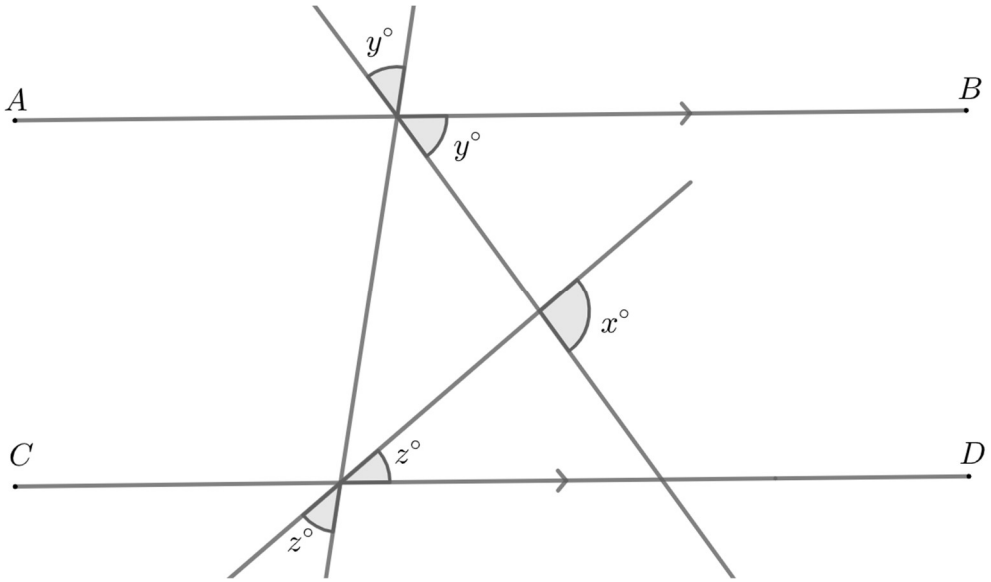
f) Make x the subject of
$y=5+\sqrt{\left(\dfrac{3x+2}{4}\right)^5}$. [2]

g) Expand and simplify:
$(a^2+b)(a^4+b^2)(a^2-b).$ [2]

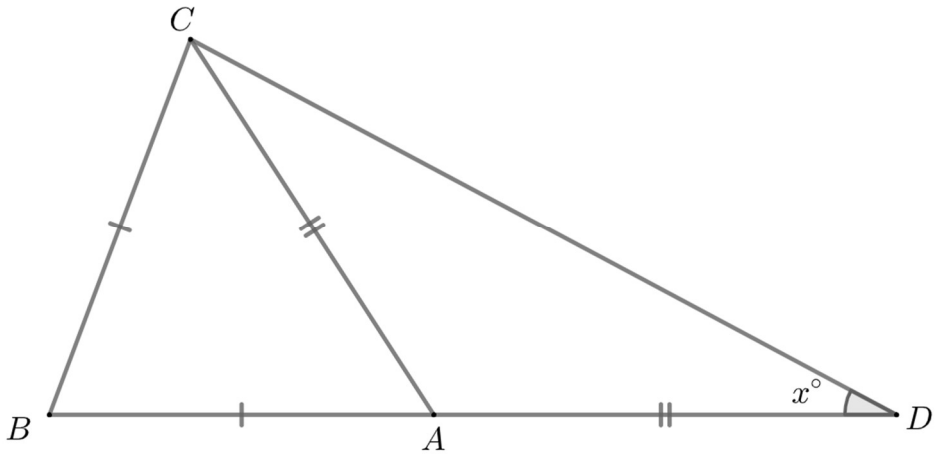
h) What is the value of
$(a^2+b)(a^4+b^2)(a^2-b)$
when $a=1$ and $b=10$? [2]

i) $\dfrac{\sqrt{17-x^3}}{4}=\sqrt[3]{\dfrac{125}{64}}.$ [3]

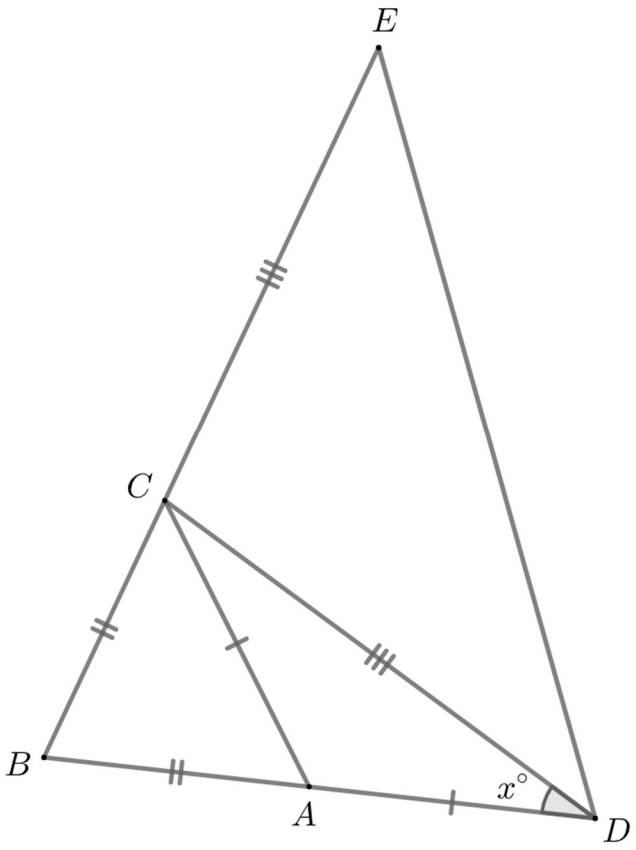
3. a) In the diagram AB and CD are parallel lines. Find the value of x. [2]



b) In the diagram $AC = AD$ and $AB = BC$.
It is given that $BD = CD$. Show that $x = \frac{180}{7}$. [3]



c) In the diagram $AC = AD$, $AB = BC$ and $CD = CE$.
It is given that $BE = DE$.
Find the value of x leaving your answer in the form $\frac{360}{n}$ where n is a whole number. [4]



4. I have a security lock that displays a four-digit code. Each of the digits can be any whole number from 0 to 9. Each digit can be changed by a "click". A "click" can increase a digit by one or decrease a digit by one.

Note that if the digit 9 is increased by a "click" then it becomes a 0. If 0 is decreased by a "click" then it becomes a 9.

For example, it would require three "clicks" to change the code 2 9 4 3 to 2 1 3 3 by "clicking" the second digit up by two and by "clicking" the third digit down by one.

a) What is the minimum number of "clicks" required to change the code 0 0 0 0 to 3 4 7 9 ? [2]

b) What is the largest number I can get after exactly five "clicks" when I start with 0 0 0 0 ? [1]

c) What is the maximum number of "clicks" that could be required to change one code to another. Give an example. [2]

d) I have forgotten the correct code that opens the lock. How many "clicks" are required to test every code, in ascending order, from 0 0 0 0 to 9 9 9 9 ? [3]

5. **Fraction 1** is

$$\frac{1}{2}.$$

Fraction 2 is

$$\frac{1}{2-\frac{1}{2}}.$$

Fraction 3 is

$$\frac{1}{2-\frac{1}{2-\frac{1}{2}}}.$$

Fraction 4 is

$$\frac{1}{2-\frac{1}{2-\frac{1}{2-\frac{1}{2}}}}$$

and so on.

a) Find the value of **Fraction 3** giving your answer as a fully simplified fraction in the form $\frac{a}{b}$. [2]

b) Likewise, find the value of **Fraction 10**. [2]

c) The difference between **Fraction** $\boldsymbol{k}$ and **Fraction** $(\boldsymbol{k+1})$ is $\frac{1}{182}$.
Find **Fraction** $(\boldsymbol{k+2})$. [3]

6. a) Amir and Beth are 10m apart and facing each other. They walk 2m towards each other. They both turn clockwise by 90 degrees. They then walk forwards for 4m. How far apart are Amir and Beth now? [3]

b) A and B are two drones hovering in the air at the same height far above the ground. A is 10m north of B.

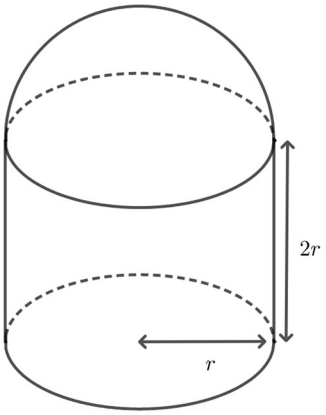
Drone A descends 10m and flies 10m west.

Drone B ascends 10m and flies 10m east.

How far apart are the drones now? [4]

7. A sphere with radius r has volume $\frac{4}{3}\pi r^3$ and surface area $4\pi r^2$.
A cylinder with height h and radius r has volume $\pi r^2 h$ and *curved* surface area $2\pi rh$.

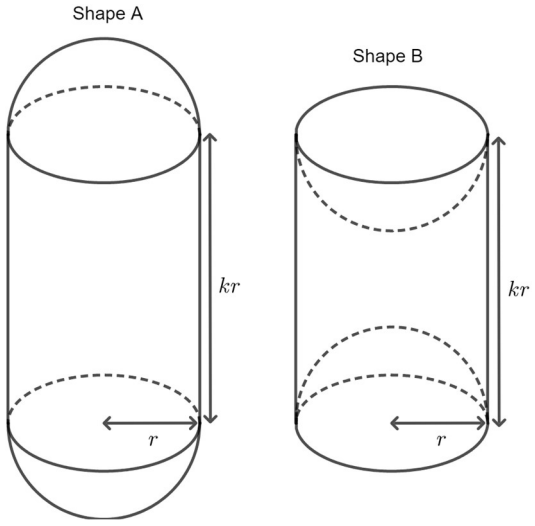
a) The shape below is a hemisphere with radius r with its circular base shared with the top of a cylinder with radius r and height $2r$. The volume of the shape is numerically equal to the surface area of the shape. Find r. [3]



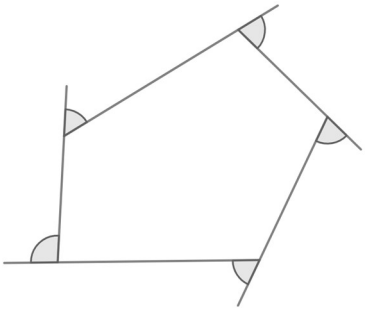
b) Shape A below on the left has twice the volume of Shape B below on the right. Shape A is a cylinder with a hemisphere added to the top and to the bottom as shown in the diagram. Shape B is a cylinder with a hemisphere removed from the top and from the bottom as shown in the diagram.

The cylinders have radius r and height kr. The hemispheres have radius r.

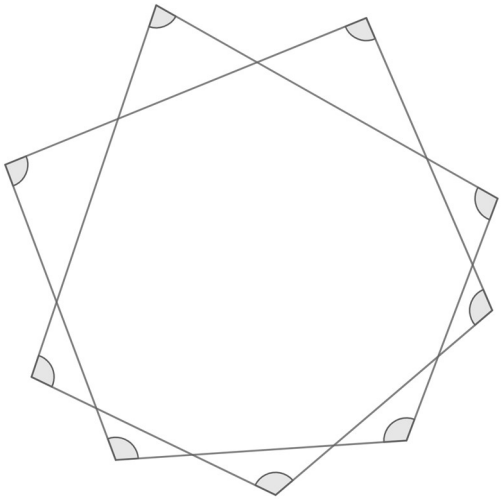
Find the value of k. [4]



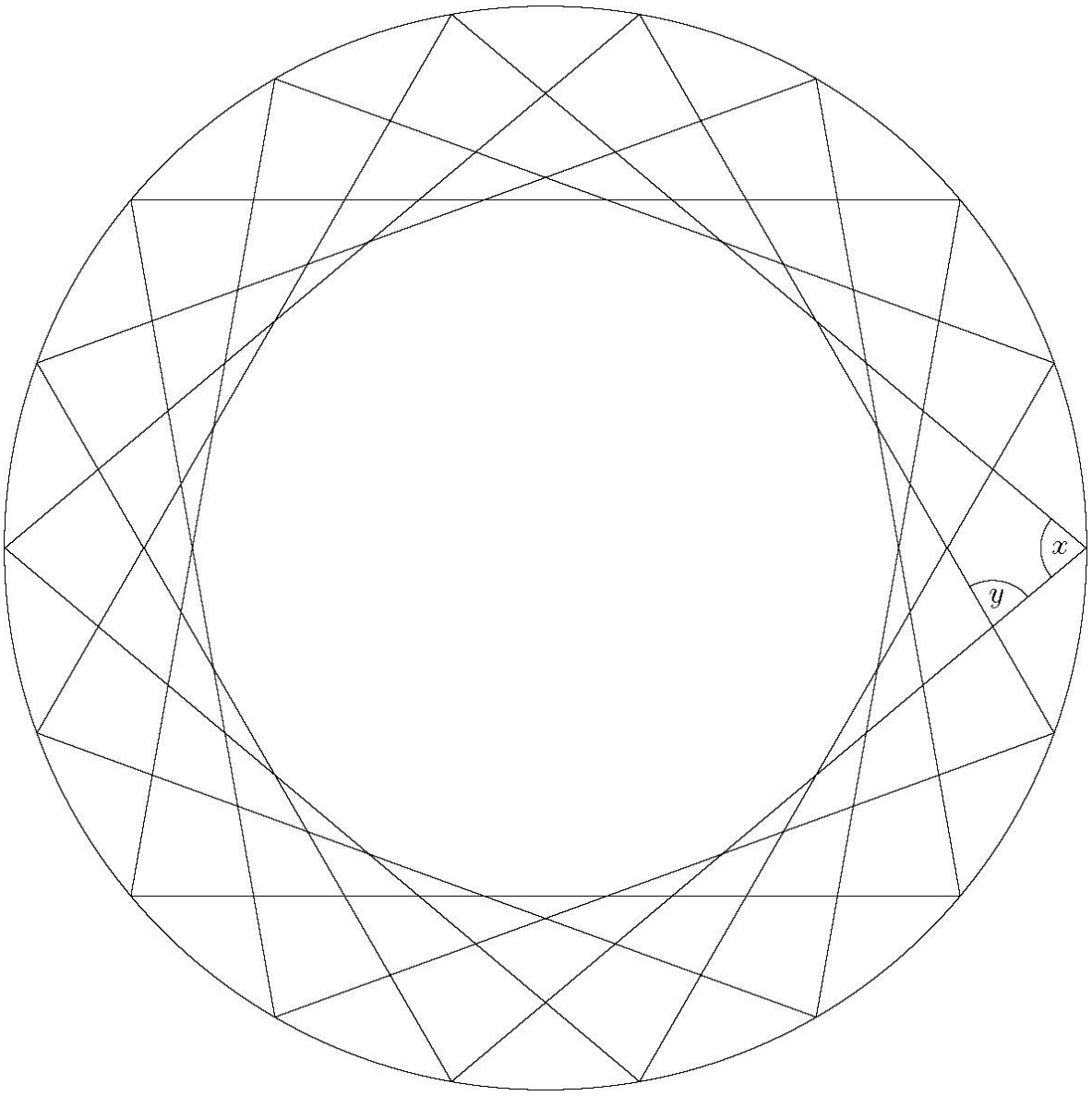
8. a) Find the sum of the five marked angles in the diagram below. [1]



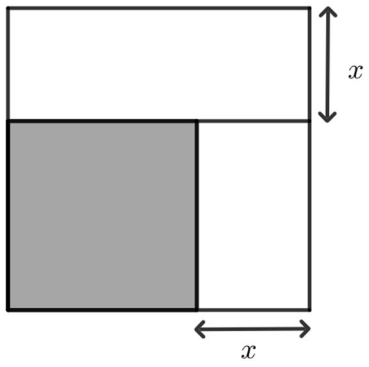
b) Find the sum of the nine marked angles in the diagram below. [2]



c) The diagram shows 18 evenly spaced points on a circle connected by line segments. The segments form a single path which winds five times around the centre of the circle before returning to its starting point. Find the size of angles x and y. [4]



9. The diagram shows a square with side length 12 that has been cut into a square and two rectangles.



By considering the total area Fred notes that

$$144 = (12 - x)^2 + 12x + x(12 - x)\,.$$

So

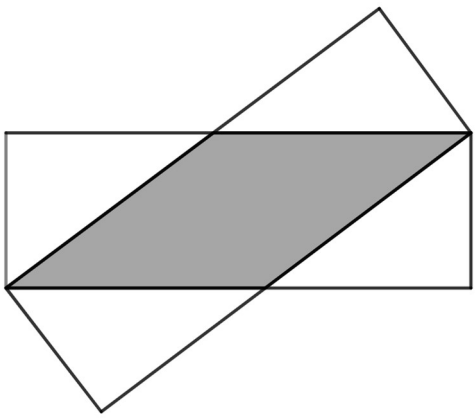
$$144 = (12 - x)^2 + 12x + 12x - x^2\,.$$

And so

$$(12 - x)^2 = 144 - 24x + x^2.$$

a) Find a similar expression for $(9 - x)^2$. [2]

b) The diagram shows two rectangles with length 9 and width 3 which touch at two corners. Find the shaded area. [4]



10. In this question you may wish to make use of the following numerical facts:

$52 = 4 \times 13 = 3 \times 17 + 1$
$170 = 13 \times 13 + 1 = 10 \times 17$

$1111 = 11 \times 101 = 30 \times 37 + 1$
$2627 = 26 \times 101 + 1 = 71 \times 37$

$1001 = 77 \times 13 = 8 \times 125 + 1$
$5500 = 423 \times 13 + 1 = 44 \times 125$
$1625 = 13 \times 125$

a) The number $N = 3 \times 170 + 5 \times 52$.

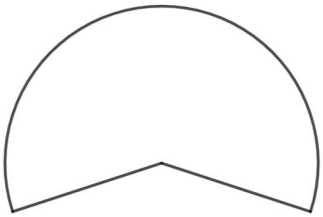
i) Find the remainder when N is divided by 13. [1]

ii) Find the remainder when N is divided by 17. [1]

b) Find a four-digit number which leaves remainder 2 when divided by 101 and leaves remainder 3 when divided by 37. [2]

c) Find a three-digit number which leaves remainder 2 when divided by 13 and leaves remainder 6 when divided by 125. [3]

11. a) A small party hat in the shape of a cone has a base radius of 3 inches and a height of 4 inches.
The hat was made by cutting out the shape shown below from a flat piece of card, and gluing the two straight edges together.



i) What is the circumference of the base of the cone? Leave your answer in terms of π. [1]

ii) What is the perimeter of the piece of card that was cut out? Leave your answer in terms of π. [2]

iii) What is the area of the piece of card that was cut out? Leave your answer in terms of π. [2]

b) The diagram below shows a cone which has a circular base with radius 60m. The shortest distance from a point at the base of the cone to the top of the cone is 120m.

Three ants want to get from a point, A, at the base of the cone to the point D that is diametrically opposite A on the base.

Alice walks around the base of the cone from A to D.

Bob walks 30m straight up the side of the cone to the point B, then turns and takes the shortest possible route across the curved surface of the cone to reach D.

Chloe walks 70m straight up the side of the cone to the point C, then turns and takes the shortest possible route across the curved surface of the cone to reach D.

All three ants walk at the same speed throughout. In what order do they arrive?
You should justify your answer and may use the fact that $3.1 < \pi < 3.2$. [5]