

13+ PAST PAPER PACK

# Winchester College 13+ Maths 2024

Complete Past Paper Pack

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Winchester College 13+ Maths. Work through this paper first.

Includes Paper Notes: overview, topics, revision tips, common mistakes.

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NAME: \_\_\_\_\_

SCHOOL: \_\_\_\_\_



WINCHESTER  
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## WINCHESTER ELECTION

### Mathematics I

Wednesday 1<sup>st</sup> May 2024

Time Allowed: 90 minutes

Total Marks: 100

Additional Information:

*CALCULATORS ARE NOT ALLOWED.*

*Write your answers in this booklet. If you need additional space, please write on sheets of A4 paper and attach them to this booklet. You should show all your working so that credit may be given for partly correct answers.*

*Diagrams are not drawn to scale.*

*Do not be discouraged if you do not finish.*

<p><b>1.</b></p>	<p>a) Find <math>0.2 \times 90</math>.</p>	<p>b) Find <math>11088 - 997</math>.</p>	<p>[1] [1]</p>
	<p>c) Find 150% of 150.</p>	<p>d) Find <math>\frac{7}{12}</math> of 108.</p>	<p>[1] [1]</p>
<p><b>2.</b></p>	<p>Kareem has a four digit passcode on his phone. Each digit is a number from 1 to 9. He can use each number more than once. He tells his friend Nina that:</p> <ul style="list-style-type: none"> <li>- The first digit is an even number.</li> <li>- The second digit is a cube number.</li> <li>- The third digit is a prime number.</li> <li>- The fourth digit is greater than (but not including) four.</li> </ul> <p>How many possible passcodes are there for Kareem's phone?</p>		<p>[4]</p>

<p><b>3.</b></p>	<p>Calculate:</p> <p>a) <math>2 + 2 \times 246 \times 5 + 6 =</math></p>	<p>b) <math>\frac{55 + 77 + 110}{33 + 88} =</math></p>	<p>[1] [1]</p>
	<p>c) <math>\sqrt{14400}</math></p>	<p>d) <math>4 \div 0.25 =</math></p>	<p>[1] [1]</p>
	<p>e) <math>2^4 \times 3^2 \times 5^4 =</math></p>	<p>f) 22% of 50% of 500 =</p>	<p>[2] [2]</p>

4. Find in the simplest form:

a)  $\frac{23}{33} - \frac{12}{33}$

b)  $2\left(\frac{2}{7} + \frac{3}{14}\right)$

[2]  
[2]

c)  $\frac{17}{38} \times \frac{57}{26} \div \frac{68}{39}$

d)  $\frac{\frac{35}{12} \times \frac{6}{7}}{\frac{1}{9} + \frac{7}{18}}$

[2]  
[3]

5.

a) Solve  $x - 5 = 18$ .

b) Solve  $6x + 9 = 10x - 19$ .

[1]

[2]

c) Solve  $\frac{10}{1 - 3x} = -5$ .

d) Solve  $\frac{32}{3 + \sqrt{2x - 1}} = 4$ .

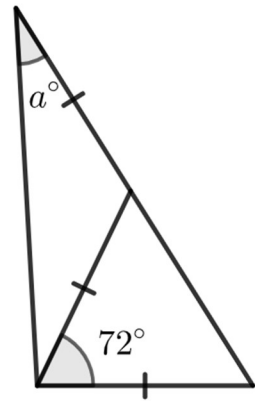
[2]

[2]

6.

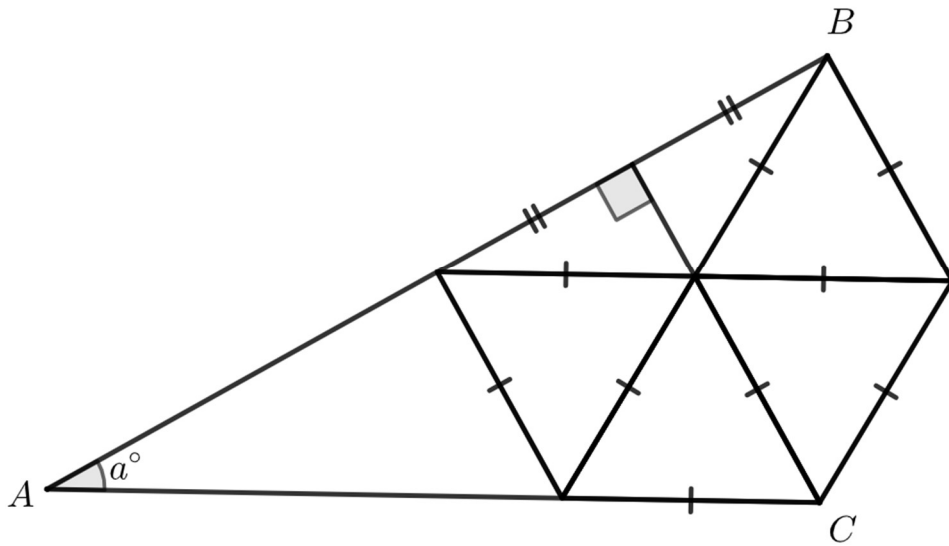
a) Find angle  $a$ .

[2]



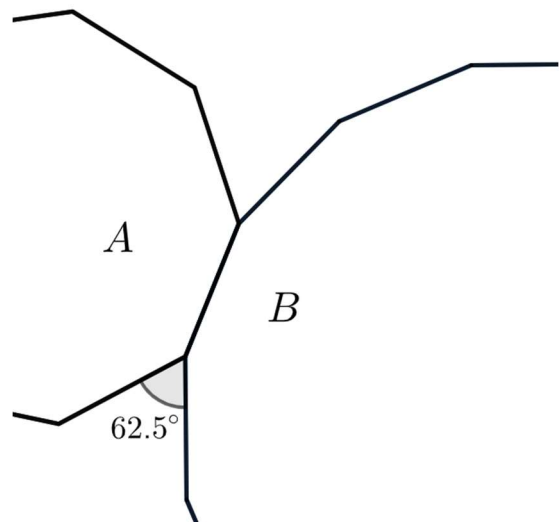
b) The diagram shows seven triangles, four of which are equilateral.  $AB$  and  $AC$  are straight lines. Find angle  $a$ .

[2]



c) The diagram below partially shows two regular polygons,  $A$  and  $B$ .  $A$  has 9 sides. How many sides has  $B$ ?

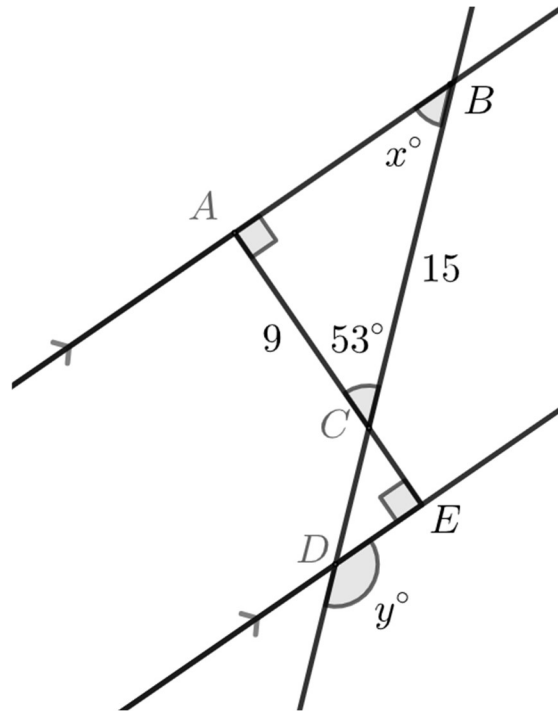
[3]



7.	$a = 4, b = 17, c = -9$ and $d = 3$ . Find the value of:  a) $\frac{3a - c}{d}$	b) $(b + c)^2 - a$	[1] [1]
	c) $ac + bd$	d) $(a + d)(a - d)$	[1] [2]
	e) $\frac{c^2 - b}{a^3}$		[2]

<p><b>8.</b></p>	<p>a) 150 can be written as a product of its prime factors as: <math>150 = 2 \times 3 \times 5^2</math>. Write 180 as a product of its prime factors.</p>	<p>b) Write the highest common factor of 150 and 180 as a product of its prime factors.</p>	<p>[2] [2]</p>
	<p>c) What is the ratio of 150 to the highest common factor of 150 and 180?</p>	<p>d) Simplify the ratio 12 : 27 : 48.</p>	<p>[2] [3]</p>
	<p>e) Three friends win a quiz. Ella correctly answers twice as many questions as Fergus, who in turn correctly answered five times as many as Gary. They split the prize of £40 in proportion to how many questions they got correct. How much does Fergus receive?</p>		<p>[3]</p>

9.



In the diagram above  $A$ ,  $B$ ,  $C$ ,  $D$  and  $E$  are points where lines intersect. The angle  $ACB$  is  $53^\circ$ , rounded to the nearest whole number.  $AC$  has length 9 and  $BC$  has length 15. The line through  $A$  and  $B$  is parallel to the line through  $D$  and  $E$ .

a) Find  $x$  and  $y$ .

[2]

b) Find the length  $AB$ .

[2]

The area of triangle  $ABC$  is nine times larger than the area of triangle  $CDE$ .

c) Find the length  $CD$ .

[2]

**10.** A list of five numbers is

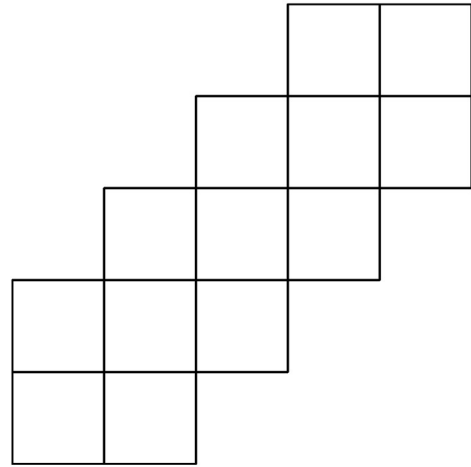
0, 2, 5, 7, 11.

Two more whole numbers,  $a$  and  $b$ , are added to this list so that both the mean and the median increase by two.

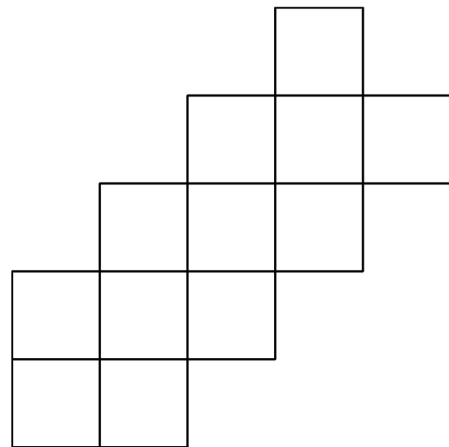
List every possible combination of  $a$  and  $b$ .

[4]

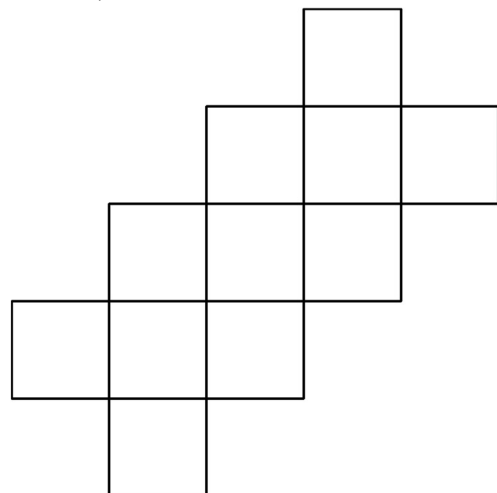
11. a) Exactly three of the thirteen squares in the shape below are to be coloured in. The resulting pattern must have rotational symmetry of order 2. In how many ways can this be done? [2]



- b) Exactly three of the twelve squares in the shape below are to be coloured in. The resulting pattern must have a line of symmetry. In how many ways can this be done? (You should give some indication as to how you arrive at your answer.) [3]



- c) Exactly three of the eleven squares in the shape are to be coloured in. The resulting pattern must have a line a symmetry. In how many ways can this be done? (You should give some indication as to how you arrive at your answer.) [3]



<p><b>12.</b></p>	<p>Taylor is at a bench and Sundip is at a tree that are 8 metres apart. Taylor runs towards the tree at 6 m/s at the same time that Sundip walks towards the bench at 2 m/s.</p> <p>a) How far are they from the bench when they meet?</p> <p>When Taylor reaches the tree she turns around and immediately runs back to the bench. When she reaches the bench she then runs back to the tree and then finally back to the bench, all the while maintaining a speed of 6 m/s.</p> <p>Likewise, when Sundip reaches the bench, he turns around immediately and walks back to the tree, then back to the bench and then back to the tree, all the while maintaining a speed of 2 m/s.</p> <p>b) How many times do Taylor and Sundip meet before Taylor finishes her journey?</p> <p>c) How does your answer to part b) change if the bench and tree are instead 800 metres apart?</p>	<p>[1]</p> <p>[4]</p> <p>[1]</p>
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<p><b>13.</b></p>	<p>A sequence is defined by a starting whole number and the following rules:</p> <ul style="list-style-type: none"> <li>• If the number is even it is divided by 2 to give the next number in the sequence.</li> <li>• If the number is odd it is added to 1 to give the next number in the sequence.</li> <li>• The sequence stops when it gets to 1.</li> </ul> <p>For example, if I start at 20, I get the 8-term sequence:</p> <p style="text-align: center;">20, 10, 5, 6, 3, 4, 2, 1.</p> <p>a) How many terms are in the sequence that starts at 21?</p> <p>b) I start with a number bigger than 1000 and create a sequence with exactly 11 terms. What was my starting number?</p> <p>c) Find a number less than 100 that results in a sequence of length 14.</p> <p>d) The first term and the third term differ by 31. What could the first term be?</p>	<p>[1]</p> <p>[2]</p> <p>[2]</p> <p>[2]</p>
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**14.** To find the number of positive factors a number has you can look at the prime factorisation of the number, raise all the powers by one, and multiply the results together.

For example, to find the number of factors of  $200 = 2^3 \times 5^2$  we calculate  
 $(3 + 1) \times (2 + 1) = 12$ .

a) Find the number of factors of 450.

[2]

A machine has a number on a display and four buttons which change the number on the display as follows:

A: increases the number by 50%

B: decreases the number by 10%

C: increases the number by 75%

D: increases the number by 96%

b) The starting number is 1000 and B is pressed three times. How many factors does the final number have?

[3]

c) The starting number is 250. A and D are pressed once each. How many factors does the final number have?

[3]

d) The starting number is 10000. Is it possible to press buttons in such a way that the final number is a whole number with more than 80 factors? (*If yes, explain how; if no, explain why not.*)

[3]

# Paper Notes: 13+ Maths Question Paper (13+ Maths Past Paper (2024))

Compiled by [SATs-Papers.co.uk](https://www.SATs-Papers.co.uk) to help you get the most from this paper.

## Overview

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This is the **Winchester College 13+ Mathematics I** entrance examination paper, sat on **Wednesday 1st May 2024**. It forms part of Winchester College's selection process for pupils entering Year 9, testing mathematical ability and problem-solving skills in a rigorous, academically demanding setting. The paper is designed to stretch the most able candidates and to identify those who can think creatively under time pressure.

Candidates are given **90 minutes** to complete a paper worth **100 marks**, and **calculators are not allowed**. The questions span a wide range of topics, from basic arithmetic and algebra to geometry, symmetry, ratio, prime factorisation, and speed-time-distance problems. The format is free-response, requiring clear working and explanations for full credit. The paper includes some multi-step problems that demand sustained reasoning and insight.

This paper is appropriate for high-achieving pupils in Year 8 preparing for competitive independent school entry. It rewards careful reading, systematic working, and the ability to tackle unfamiliar or novel problem types. The level of difficulty is noticeably higher than typical 11+ material, reflecting the more selective and academic nature of 13+ entrance exams at top independent schools.

## How this paper is organised

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The paper is divided into **fourteen questions**, numbered 1 to 14, with mark allocations ranging from single-mark starter calculations to multi-part investigations worth up to 4 marks. Question 1 begins with straightforward arithmetic (finding products, differences, percentages, and fractions), allowing pupils to settle into the paper. Questions then increase in complexity and conceptual demand, covering algebra, geometry, prime factorisation, ratio, symmetry, sequences, and speed-time-distance problems.

Each question is broken into labelled parts (a, b, c, and so on), and marks are shown in square brackets beside each part. The rubric instructs candidates to show all working so that partial credit can be awarded, and diagrams are noted as not to scale. The final questions (12, 13, and 14) require extended reasoning and multi-step problem-solving, testing pupils' ability to organise their work and communicate their methods clearly.

With **90 minutes** for **100 marks**, candidates have roughly 54 seconds per mark on average. However, some questions are much quicker to answer than others, so

effective time management and prioritisation are essential. Pupils are advised not to be discouraged if they do not finish, a sign that the paper is designed to challenge and differentiate across a wide range of ability.

## Topics covered

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- Arithmetic with decimals, percentages, fractions, and order of operations (e.g.  $0.2 \times 90$ , 150% of 150,  $4 \div 0.25$ )
- Combinatorial counting and systematic reasoning (passcode problem with constraints on digits)
- Simplification and operations with fractions, including addition, subtraction, multiplication, and division of mixed numbers
- Linear and fractional equations, including solving equations with radicals and rational expressions
- Angle properties of triangles, parallel lines, alternate angles, and properties of isosceles and equilateral triangles
- Interior and exterior angles of regular polygons, including deduction from partial diagrams
- Substitution of negative integers into algebraic expressions and difference-of-squares identities
- Prime factorisation, highest common factors, ratio simplification, and proportion problems
- Pythagoras' theorem applied to right-angled triangles, rounding, and similar triangles with area scale factors
- Mean and median of data sets, including systematic enumeration of integer solutions under constraints
- Rotational and reflective symmetry in grid patterns, requiring careful counting and explanation
- Speed-time-distance problems with multiple legs and relative motion, including extending patterns to large scales
- Number sequences defined by rules (halving even numbers, incrementing odd numbers), working forwards and backwards
- Counting factors using prime factorisation, and manipulating numbers through percentage changes to maximise factor counts

## How to use this paper for revision

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- Revise the **order of operations** (BIDMAS / BODMAS) carefully, particularly when multiplication, division, addition, and subtraction appear in the same expression without brackets.
- Practise **fraction arithmetic** without a calculator, including converting mixed numbers to improper fractions, finding common denominators, and simplifying results to lowest terms.
- Review **angle properties** in parallel lines (alternate angles, corresponding angles) and triangles (sum of angles, exterior angle theorem, isosceles triangles).
- Strengthen your understanding of **prime factorisation** and how to use it to find highest common factors, lowest common multiples, and the number of factors a number has.
- Work through problems involving **ratio and proportion**, including dividing quantities in given ratios and scaling factors in similar shapes.
- Develop systematic approaches to **combinatorial counting** and **case-by-case enumeration**, especially when multiple constraints apply.
- Practise **speed-time-distance** problems, including relative motion, meeting times, and problems where objects reverse direction multiple times.

## Common mistakes to avoid

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- Misapplying order of operations by adding before multiplying, especially in expressions like  $2 + 2 \times 246 \times 5 + 6$ , leading to wildly incorrect answers.
- Forgetting to simplify fractions to their lowest terms, or making arithmetic errors when finding common denominators in multi-step fraction calculations.
- Confusing **alternate angles** with **corresponding angles**, or failing to spot that lines are parallel when deducing angle relationships.
- Incorrectly applying the formula for the number of factors; remember to add 1 to each prime power exponent before multiplying, not after.
- Rushing combinatorial problems and missing cases, or double-counting configurations that are equivalent under symmetry operations.
- Misreading constraints in word problems (e.g. 'greater than but not including four' means 5, 6, 7, 8, 9, not 4 onwards).

## Exam technique

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Start by skimming the entire paper to identify questions you can answer quickly and confidently. Complete these first to bank marks and build momentum. Questions 1 and 3 are designed as accessible entry points, so tackle them early to settle your nerves. Then move systematically through the paper, spending more time on multi-mark questions (such as questions 10, 11, 12, and 14) that reward extended reasoning and careful explanation.

For questions involving algebra, geometry, or sequences, **show all your working clearly**. Winchester's marking scheme awards partial credit for correct methods, even if your final answer is wrong. Write out each step, label diagrams, and annotate your reasoning so that examiners can follow your thinking. If you get stuck, move on and return later rather than wasting time on a single question.

Manage your time by aiming to reach question 10 within the first hour, leaving 30 minutes for the final, more demanding problems. If you find yourself short on time, prioritise parts (a) and (b) of later questions, which are often more straightforward than parts (c) and (d). Remember that the rubric says 'Do not be discouraged if you do not finish', so aim for quality over speed. A well-explained solution to three-quarters of the paper is more valuable than rushed, incomplete attempts at every question.

## What to revise alongside this paper

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Pupils should revise **algebraic manipulation**, including factorising quadratics, expanding brackets, and rearranging formulae, as these skills underpin many of the algebra questions. Strengthen your grasp of **Pythagoras' theorem** and **trigonometry** (sine, cosine, tangent), as these are often tested at 13+ level alongside angle and similarity problems. Review **sequences and series**, including arithmetic and geometric progressions, and practise working backwards from later terms to find starting values.

For geometry, extend your knowledge to include **circle theorems**, **congruence**, and **transformations** (reflection, rotation, enlargement), as these topics frequently appear in 13+ papers. Work through problems involving **simultaneous equations**, both linear and one linear with one quadratic, to build confidence in multi-step algebraic reasoning. Finally, deepen your understanding of **number theory** by exploring divisibility rules, modular arithmetic, and the properties of primes, squares, and cubes.

For those aiming at the most competitive independent schools, consider tackling past papers from the **UKMT Junior Mathematical Challenge** or **Intermediate Mathematical Olympiad**, which test similar problem-solving skills and mathematical creativity. These resources will help you develop the resilience and insight needed to tackle unfamiliar or multi-step problems under timed conditions.

## Key terms

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**Prime factorisation, Highest common factor (HCF), Lowest common multiple (LCM), Order of operations (BIDMAS/BODMAS), Alternate angles, Corresponding angles, Isosceles triangle, Equilateral triangle, Interior angle (of a polygon), Exterior angle (of a polygon), Rotational symmetry, Line of symmetry, Similar triangles, Scale factor, Pythagoras' theorem, Mean and median, Speed-time-distance, Relative motion, Combinatorial counting, Systematic enumeration**

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NAME: \_\_\_\_\_

SCHOOL: \_\_\_\_\_



WINCHESTER  
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## WINCHESTER ELECTION

### Mathematics II

Tuesday 30<sup>th</sup> April 2024

Time Allowed: 90 minutes

Total Marks: 100

Additional Information:

*CALCULATORS ARE NOT ALLOWED.*

*Write your answers in this booklet. If you need additional space, please write on sheets of A4 paper and attach them to this booklet. You should show all your working so that credit may be given for partly correct answers.*

*Diagrams are not drawn to scale.*

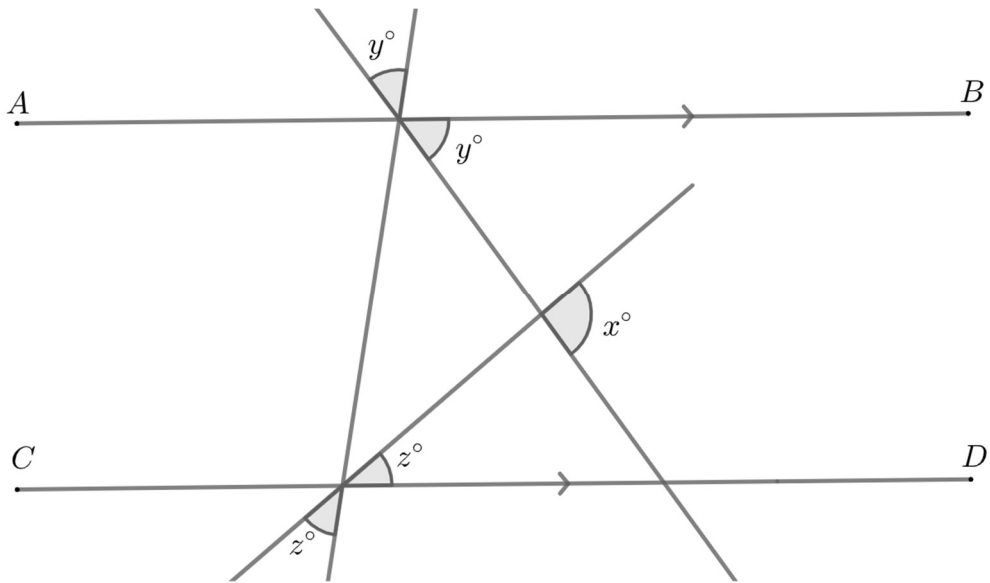
*Do not be discouraged if you do not finish.*

1.	Evaluate: a) $121 + 8 \times 121 + 2(121)$ .	b) $5 \times 2^3 \times 3 \times 2 \times 5^2$ .	[1] [1]
	c) 5% of 25% of 50% of 3200.	d) $\frac{\sqrt[3]{64}^2}{\sqrt{4^3}}$ .	[1] [2]
	e) $\frac{\sqrt{1.44}}{\sqrt{0.04}}$ .	f) $\frac{(-27)^4}{9^5}$ .	[2] [2]
	g) Determine $n$ , where $2^n = 2^3 + 2^3 + 2^3 + 2^3$ .		[2]
	h) What fraction is halfway between $\frac{1}{8}$ and $\frac{1}{9}$ .		[2]

<p>2. Solve:</p> <p>a) <math>\frac{81 - x}{2x + 6} = 3.</math></p>	<p>b) <math>3(2x + 5) - 2(6 - 2x) = 43.</math></p> <p>[2] [2]</p>
<p>c) <math>\frac{5}{4 - 4x} = \frac{2}{x + 12}.</math></p>	<p>d) Solve <math>\frac{3x^3}{8} + 25 = 1.</math></p> <p>[2] [2]</p>
<p>e) Expand and simplify <math>2a(2b + 3a) - 3b(3b - 2a).</math></p>	<p>f) Make <math>x</math> the subject of <math>y = 5 + \sqrt{\left(\frac{3x + 2}{4}\right)^5}.</math></p> <p>[2] [2]</p>
<p>g) Expand and simplify: <math>(a^2 + b)(a^4 + b^2)(a^2 - b).</math></p>	<p>h) What is the value of <math>(a^2 + b)(a^4 + b^2)(a^2 - b)</math> when <math>a = 1</math> and <math>b = 10</math>?</p> <p>[2] [2]</p>
<p>i) <math>\frac{\sqrt{17 - x^3}}{4} = \sqrt[3]{\frac{125}{64}}.</math></p>	<p>[3]</p>

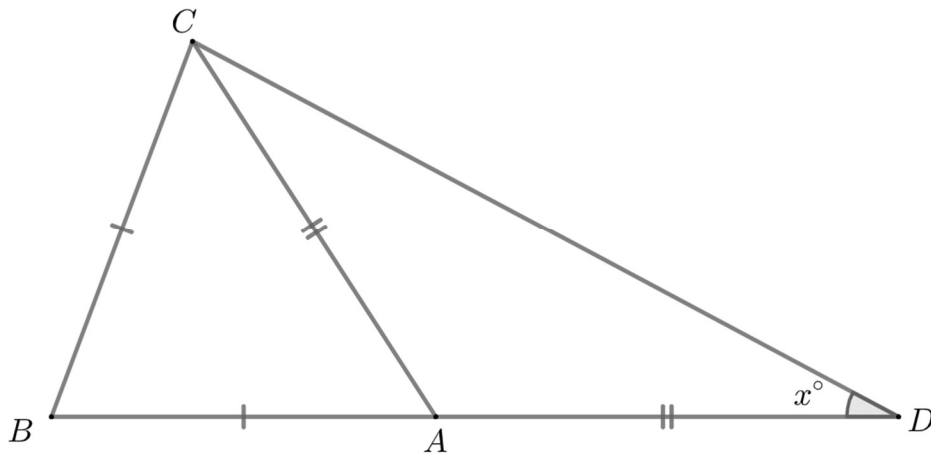
3. a) In the diagram  $AB$  and  $CD$  are parallel lines. Find the value of  $x$ .

[2]



b) In the diagram  $AC = AD$  and  $AB = BC$ .  
It is given that  $BD = CD$ . Show that  $x = \frac{180}{7}$ .

[3]

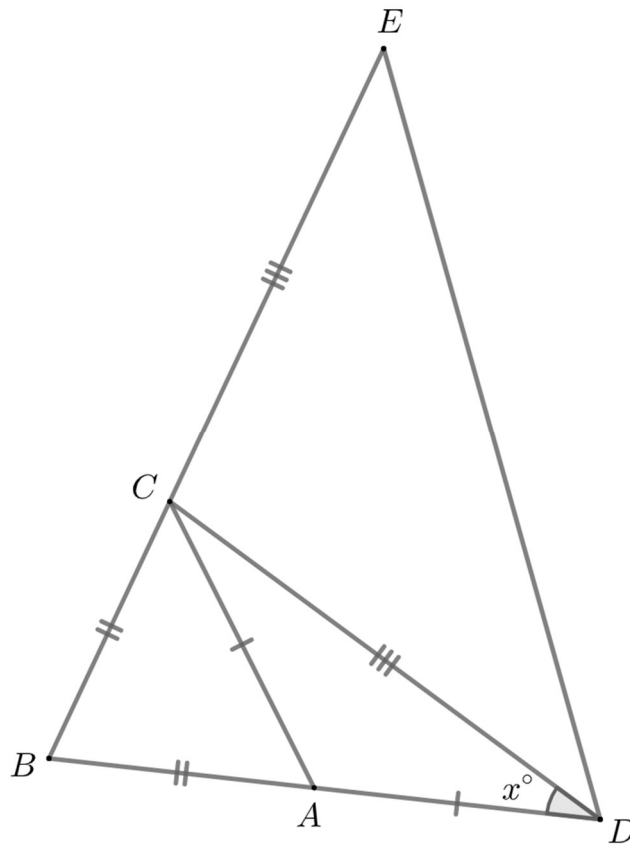


c) In the diagram  $AC = AD$ ,  $AB = BC$  and  $CD = CE$ .

It is given that  $BE = DE$ .

Find the value of  $x$  leaving your answer in the form  $\frac{360}{n}$  where  $n$  is a whole number.

[4]



4. I have a security lock that displays a four-digit code. Each of the digits can be any whole number from 0 to 9. Each digit can be changed by a “click”. A “click” can increase a digit by one or decrease a digit by one.

Note that if the digit 9 is increased by a “click” then it becomes a 0. If 0 is decreased by a “click” then it becomes a 9.

For example, it would require three “clicks” to change the code  $\boxed{2}\boxed{9}\boxed{4}\boxed{3}$  to  $\boxed{2}\boxed{1}\boxed{3}\boxed{3}$  by “clicking” the second digit up by two and by “clicking” the third digit down by one.

- a) What is the minimum number of “clicks” required to change the code  $\boxed{0}\boxed{0}\boxed{0}\boxed{0}$  to  $\boxed{3}\boxed{4}\boxed{7}\boxed{9}$ ? [2]

- b) What is the largest number I can get after exactly five “clicks” when I start with  $\boxed{0}\boxed{0}\boxed{0}\boxed{0}$ ? [1]

- c) What is the maximum number of “clicks” that could be required to change one code to another. Give an example. [2]

- d) I have forgotten the correct code that opens the lock. How many “clicks” are required to test every code, in ascending order, from  $\boxed{0}\boxed{0}\boxed{0}\boxed{0}$  to  $\boxed{9}\boxed{9}\boxed{9}\boxed{9}$ ? [3]

5. **Fraction 1** is

$$\frac{1}{2}.$$

**Fraction 2** is

$$\frac{1}{2 - \frac{1}{2}}.$$

**Fraction 3** is

$$\frac{1}{2 - \frac{1}{2 - \frac{1}{2}}}.$$

**Fraction 4** is

$$\frac{1}{2 - \frac{1}{2 - \frac{1}{2 - \frac{1}{2}}}}.$$

and so on.

a) Find the value of **Fraction 3** giving your answer as a fully simplified fraction in the form  $\frac{a}{b}$ . [2]

b) Likewise, find the value of **Fraction 10**. [2]

c) The difference between **Fraction  $k$**  and **Fraction  $(k + 1)$**  is  $\frac{1}{182}$ . Find **Fraction  $(k + 2)$** . [3]

6. a) Amir and Beth are 10m apart and facing each other. They walk 2m towards each other. They both turn clockwise by 90 degrees. They then walk forwards for 4m. How far apart are Amir and Beth now?

[3]

b) A and B are two drones hovering in the air at the same height far above the ground. A is 10m north of B.

Drone A descends 10m and flies 10m west.

Drone B ascends 10m and flies 10m east.

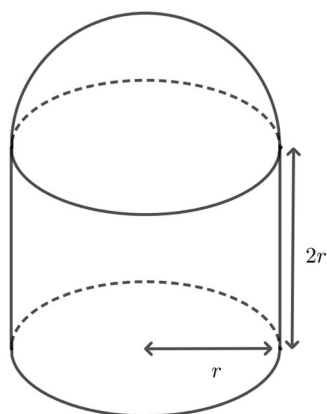
How far apart are the drones now?

[4]

7. A sphere with radius  $r$  has volume  $\frac{4}{3}\pi r^3$  and surface area  $4\pi r^2$ .  
 A cylinder with height  $h$  and radius  $r$  has volume  $\pi r^2 h$  and *curved* surface area  $2\pi r h$ .

a) The shape below is a hemisphere with radius  $r$  with its circular base shared with the top of a cylinder with radius  $r$  and height  $2r$ . The volume of the shape is numerically equal to the surface area of the shape. Find  $r$ .

[3]

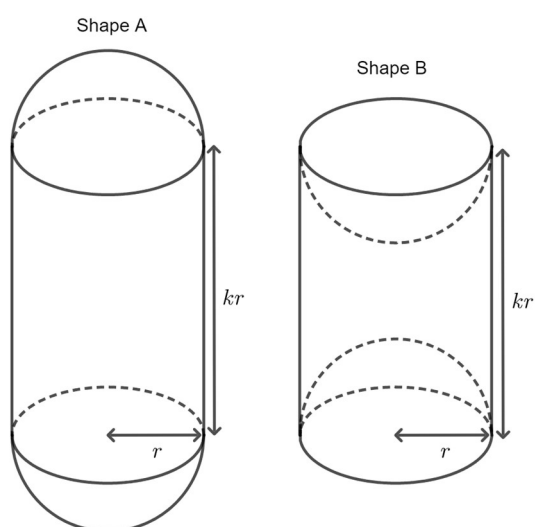


b) Shape A below on the left has twice the volume of Shape B below on the right. Shape A is a cylinder with a hemisphere added to the top and to the bottom as shown in the diagram. Shape B is a cylinder with a hemisphere removed from the top and from the bottom as shown in the diagram.

The cylinders have radius  $r$  and height  $kr$ . The hemispheres have radius  $r$ .

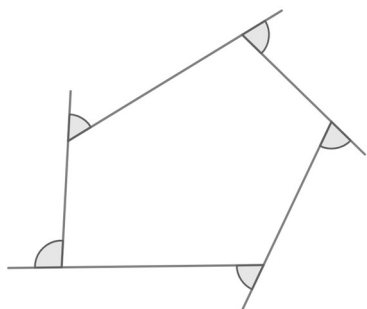
Find the value of  $k$ .

[4]



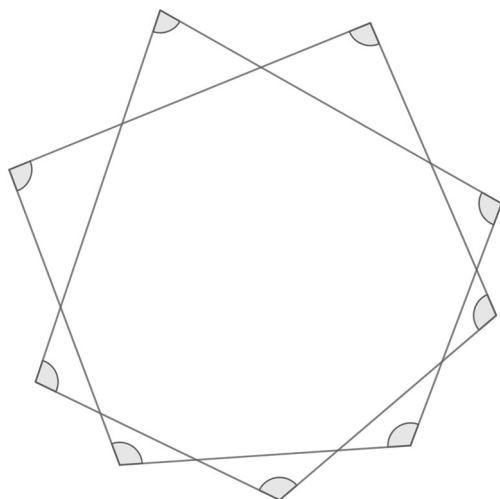
8. a) Find the sum of the five marked angles in the diagram below.

[1]



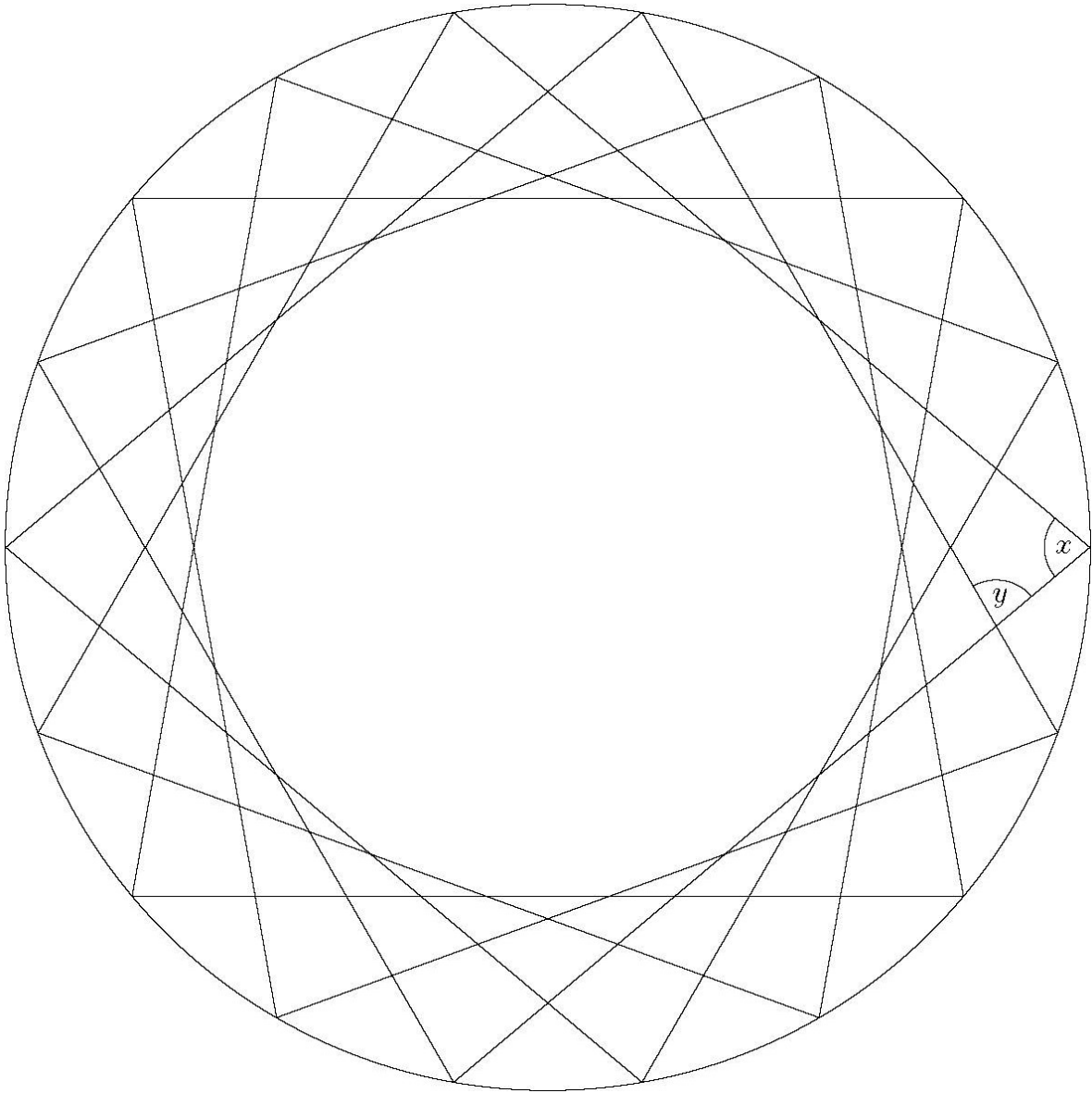
b) Find the sum of the nine marked angles in the diagram below.

[2]

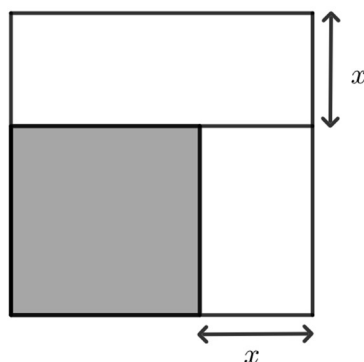


c) The diagram shows 18 evenly spaced points on a circle connected by line segments. The segments form a single path which winds five times around the centre of the circle before returning to its starting point. Find the size of angles  $x$  and  $y$ .

[4]



9. The diagram shows a square with side length 12 that has been cut into a square and two rectangles.



By considering the total area Fred notes that

$$144 = (12 - x)^2 + 12x + x(12 - x).$$

So

$$144 = (12 - x)^2 + 12x + 12x - x^2.$$

And so

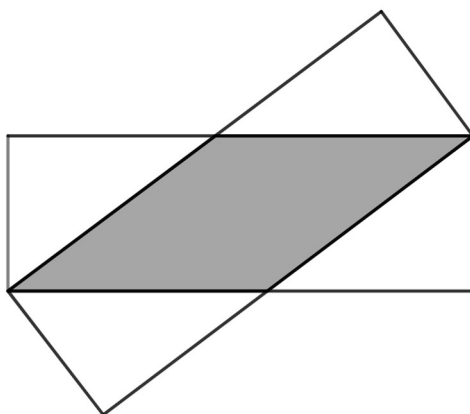
$$(12 - x)^2 = 144 - 24x + x^2.$$

- a) Find a similar expression for  $(9 - x)^2$ .

[2]

- b) The diagram shows two rectangles with length 9 and width 3 which touch at two corners. Find the shaded area.

[4]



10. In this question you may wish to make use of the following numerical facts:

$$52 = 4 \times 13 = 3 \times 17 + 1$$

$$170 = 13 \times 13 + 1 = 10 \times 17$$

$$1111 = 11 \times 101 = 30 \times 37 + 1$$

$$2627 = 26 \times 101 + 1 = 71 \times 37$$

$$1001 = 77 \times 13 = 8 \times 125 + 1$$

$$5500 = 423 \times 13 + 1 = 44 \times 125$$

$$1625 = 13 \times 125$$

a) The number  $N = 3 \times 170 + 5 \times 52$ .

i) Find the remainder when  $N$  is divided by 13.

[1]

ii) Find the remainder when  $N$  is divided by 17.

[1]

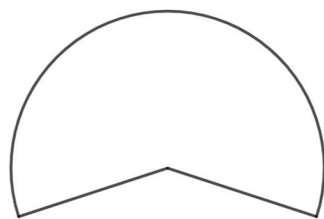
b) Find a four-digit number which leaves remainder 2 when divided by 101 and leaves remainder 3 when divided by 37.

[2]

c) Find a three-digit number which leaves remainder 2 when divided by 13 and leaves remainder 6 when divided by 125.

[3]

11. a) A small party hat in the shape of a cone has a base radius of 3 inches and a height of 4 inches.  
The hat was made by cutting out the shape shown below from a flat piece of card, and gluing the two straight edges together.



i) What is the circumference of the base of the cone? Leave your answer in terms of  $\pi$ . [1]

ii) What is the perimeter of the piece of card that was cut out? Leave your answer in terms of  $\pi$ . [2]

iii) What is the area of the piece of card that was cut out? Leave your answer in terms of  $\pi$ . [2]

b) The diagram below shows a cone which has a circular base with radius 60m. The shortest distance from a point at the base of the cone to the top of the cone is 120m.

Three ants want to get from a point,  $A$ , at the base of the cone to the point  $D$  that is diametrically opposite  $A$  on the base.

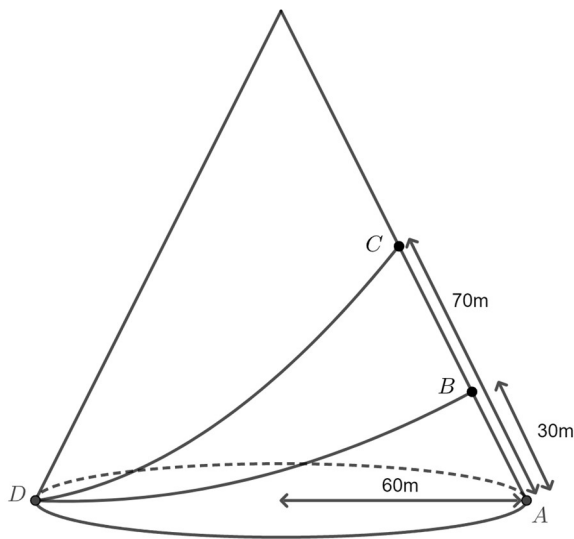
Alice walks around the base of the cone from  $A$  to  $D$ .

Bob walks 30m straight up the side of the cone to the point  $B$ , then turns and takes the shortest possible route across the curved surface of the cone to reach  $D$ .

Chloe walks 70m straight up the side of the cone to the point  $C$ , then turns and takes the shortest possible route across the curved surface of the cone to reach  $D$ .

All three ants walk at the same speed throughout. In what order do they arrive?  
You should justify your answer and may use the fact that  $3.1 < \pi < 3.2$ .

[5]





# Paper Notes: 13+ Maths Question Paper (13+ Maths Past Paper (2024))

Compiled by [SATs-Papers.co.uk](https://www.SATs-Papers.co.uk) to help you get the most from this paper.

## Overview

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This is the **Winchester College Mathematics II** paper, part of the school's **13+ entrance examination** sat in April 2024. It is designed to assess candidates for entry into Year 9 at Winchester College, one of England's most academically selective independent schools. The paper tests advanced mathematical reasoning and problem-solving skills well beyond the typical Year 8 curriculum.

The examination comprises **100 marks** to be completed in **90 minutes**, with **calculators not permitted**. Questions range from straightforward arithmetic and algebra to complex geometry, number theory, and spatial reasoning. The paper is divided into **11 numbered problems**, many with multiple sub-parts, and rewards clear working and logical exposition as much as correct final answers.

This paper suits students preparing for highly competitive 13+ entrance exams at independent schools, particularly those targeting institutions with a strong mathematical tradition. The difficulty level is substantial: several questions require sophisticated insight, and the rubric explicitly tells candidates not to be discouraged if they do not finish, signalling that full completion is neither expected nor necessary for a strong performance.

## How this paper is organised

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The paper is organised into **11 main questions**, numbered 1 to 11, with a total of **100 marks** distributed across approximately 50 individual sub-parts. Question 1 consists of eight short computation and manipulation tasks (parts a to h), each worth 1 or 2 marks, designed to test fluency in arithmetic, percentages, surds, and indices. Question 2 similarly contains nine algebraic problems (parts a to i), ranging from linear equations to rearranging formulae and cube roots, each worth 2 or 3 marks.

Questions 3 to 11 are longer, multi-step problems. Question 3 focuses on **angle geometry** with three parts totalling 9 marks. Question 4 explores a combinatorial puzzle about a four-digit security lock (8 marks total). Question 5 investigates a recursive fraction sequence (7 marks). Questions 6 and 7 involve **spatial reasoning and 3D geometry**, including distance problems and volume-surface area relationships (7 marks each). Question 8 tests angle sums in polygons and star patterns (7 marks). Question 9 uses algebraic identities derived from area decompositions (6 marks).

Question 10 applies modular arithmetic and the Chinese Remainder Theorem (7 marks). Question 11, the final problem, concerns cone geometry and path optimisation on curved surfaces (10 marks).

The layout is clear, with diagrams provided where needed (though noted as not to scale). Candidates are instructed to show all working, as partial credit is awarded for correct intermediate steps even if the final answer is incorrect. The time pressure is considerable: 90 minutes for 100 marks implies less than two minutes per mark, though many multi-mark questions require extended reasoning.

## Topics covered

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- Arithmetic operations with brackets and order of operations (BIDMAS)
- Simplification and manipulation of powers, including prime factorisation and index laws
- Percentages of percentages and compound percentage calculations
- Surds and cube roots, including rationalisation and simplification of nested roots
- Linear equations and equations involving fractions and algebraic fractions
- Rearranging formulae to make a variable the subject, including expressions under radicals and powers
- Algebraic expansion and factorisation, including products of binomials and trinomials
- Angle geometry in parallel lines, isosceles triangles, and compound figures with multiple equal sides
- Combinatorial reasoning and optimisation problems involving discrete operations (e.g. the security lock puzzle)
- Recursive sequences and continued fractions, requiring pattern recognition and algebraic manipulation
- 3D coordinate geometry and Pythagorean theorem in three dimensions
- Volume and surface area of spheres, hemispheres, and cylinders, including composite solids
- Angle sums in polygons and star polygons, exterior angles, and properties of cyclic figures
- Algebraic identities derived from geometric area dissections
- Modular arithmetic and the Chinese Remainder Theorem applied to finding numbers with given remainders
- Cone geometry, including slant height, arc length, sector area, and geodesic paths on curved surfaces

## How to use this paper for revision

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- Practise **mental arithmetic** and estimation skills, as no calculator is allowed. Rapid fluency with squares, cubes, square roots, and simple fractions will save valuable time on Questions 1 and 2.
- Revise **index laws** thoroughly, including negative and fractional powers. Many sub-parts in the opening questions test these rules in isolation and in combination.
- Work through past papers on **angle chasing** in complex geometric figures. Question 3 requires careful labelling of angles and systematic use of isosceles triangle properties and parallel line theorems.
- Study examples of **problem-solving puzzles** that involve sequences of operations or constrained counting. Question 4 on the security lock rewards clear reasoning and exhaustive case analysis.
- Familiarise yourself with **recursive definitions** and continued fractions. Question 5 is unusual and requires confidence in working backwards from a pattern.
- Review **3D Pythagoras** and coordinate geometry in three dimensions. Questions 6 and 7 test spatial visualisation and the ability to break a 3D problem into 2D slices.
- Master the formulae for **volumes and surface areas** of spheres, cones, and cylinders. Question 7 and Question 11 require fluency with these, often in combination or with composite shapes.
- Practise **modular arithmetic** and the Chinese Remainder Theorem if your syllabus covers it. Question 10 provides numerical facts to guide you, but understanding the underlying theory is essential for selecting the right combination.

## Common mistakes to avoid

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- Forgetting to **show working** for multi-mark questions. Even if the final answer is wrong, partial credit is often available for correct intermediate steps, so write down every calculation.
- Misapplying **BIDMAS** when brackets and powers are nested. Question 1(a) and similar items penalise students who compute left-to-right without respecting order of operations.
- Losing track of **negative signs** when expanding or simplifying algebraic expressions. Question 2(e) and similar parts require careful attention to signs in front of brackets.
- Overlooking the **wraparound behaviour** in the security lock problem (Question 4), where incrementing 9 gives 0 and decrementing 0 gives 9. Many students assume all clicks increase or decrease within the range 0-9.
- Confusing **volume and surface area** formulae for 3D shapes. Question 7 explicitly uses both, and mixing them up will lead to an incorrect equation and wrong value of the variable.
- Misidentifying **angle types** in complex figures. In Question 3, students often fail to spot that certain triangles are isosceles or that angles are alternate or corresponding, leading to incorrect equations.
- Not using the **given numerical facts** in Question 10. The problem provides key equalities to guide modular arithmetic reasoning; ignoring them makes the question far harder than intended.

## Exam technique

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Begin by reading the entire paper quickly to identify which questions are within your comfort zone and which are more challenging. Questions 1 and 2 contain many short sub-parts worth 1 or 2 marks each; aim to complete these efficiently to bank marks early. If a calculation becomes messy or uncertain, move on rather than spending excessive time on a single mark.

For longer problems (Questions 3 onwards), **read each part carefully** and annotate diagrams with known angles or lengths as you work. In geometry questions, label all derived angles immediately to avoid confusion later. In multi-step problems such as Question 11, break the problem into smaller tasks (e.g. first find the slant height, then the arc length, then apply Pythagoras on the flattened cone surface). Write each intermediate result clearly, as partial marks are available even if the final answer is incorrect.

Manage your time actively. With 90 marks for 100 marks, you have slightly under two minutes per mark on average, but some marks (e.g. in Question 1) should take far less, leaving more time for the 4- or 5-mark parts in Questions 8, 9, 10, and 11. If you find yourself stuck on a 2-mark question for more than four or five minutes, flag it and move on. Return at the end if time permits. Finally, **do not be discouraged** if you cannot complete the paper; the rubric itself acknowledges that full completion is rare, and strong candidates will still perform well by maximising marks on accessible questions.

## What to revise alongside this paper

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To prepare thoroughly for this paper, students should revise **advanced algebraic manipulation**, including factorisation, completing the square, and rearranging complex formulae. Many of the algebra questions in Question 2 assume fluency with these techniques. Work through problems involving **nested surds and fractional indices**, as Question 1(d) and Question 2(i) test these skills directly.

In geometry, practice **angle chasing** in figures with multiple isosceles triangles, parallel lines, and cyclic quadrilaterals. Question 3 and Question 8 require systematic reasoning with angles, and students who struggle with these should revisit Euclidean geometry proofs. For 3D problems, study **coordinate geometry in three dimensions**, including distance formulae and the use of Pythagoras in multiple planes, as tested in Questions 6 and 7.

Finally, explore **number theory** topics such as modular arithmetic, divisibility rules, and the Chinese Remainder Theorem if your syllabus includes them. Question 10 assumes familiarity with these ideas, and students who have not encountered them should seek out introductory resources. For the most challenging problems, such as Question 11(b), read about **geodesics on cones** and the technique of 'unrolling' a cone into a flat sector to find shortest paths, a topic that blends geometry with trigonometry and often appears in mathematical olympiads.

## Key terms

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**BIDMAS (order of operations), Index laws (powers and roots), Surds and cube roots, Linear equations, Rearranging formulae, Isosceles triangle, Alternate angles, Corresponding angles, Exterior angle of a polygon, Volume and surface area of a sphere, Volume and surface area of a cylinder, Slant height of a cone, Sector area and arc length, Modular arithmetic, Chinese Remainder Theorem, Pythagorean theorem in 3D**

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