NAME: _____

SCHOOL: _____

WINCHESTER
COLLEGE

# WINCHESTER ELECTION

## Mathematics I

### Wednesday 1st May 2024

Time Allowed: 90 minutes

Total Marks: 100

Additional Information:

*CALCULATORS ARE NOT ALLOWED.*

*Write your answers in this booklet. If you need additional space, please write on sheets of A4 paper and attach them to this booklet. You should show all your working so that credit may be given for partly correct answers.*

*Diagrams are not drawn to scale.*

*Do not be discouraged if you do not finish.*

| | | | |
|---|---|---|---|
| **1.** | a) Find $0.2 \times 90$. | b) Find $11088 - 997$. | [1]<br>[1] |
| | c) Find 150% of 150. | d) Find $\frac{7}{12}$ of 108. | [1]<br>[1] |
| **2.** | Kareem has a four digit passcode on his phone. Each digit is a number from 1 to 9. He can use each number more than once. He tells his friend Nina that:<br>- The first digit is an even number.<br>- The second digit is a cube number.<br>- The third digit is a prime number.<br>- The fourth digit is greater than (but not including) four.<br><br>How many possible passcodes are there for Kareem's phone? | | [4] |

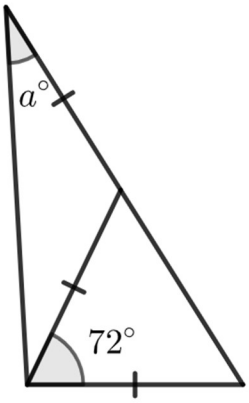
| | | | |
|---|---|---|---|
| **3.** | Calculate:<br>a) $2 + 2 \times 246 \times 5 + 6 =$ | b) $\frac{55 + 77 + 110}{33 + 88} =$ | [1]<br>[1] |
| | c) $\sqrt{14400}$ | d) $4 \div 0.25 =$ | [1]<br>[1] |
| | e) $2^4 \times 3^2 \times 5^4 =$ | f) 22% of 50% of 500 = | [2]<br>[2] |

| | | | |
|---|---|---|---|
| **4.** | Find in the simplest form:<br><br>a) $\frac{23}{33} - \frac{12}{33}$ | b) $2\left(\frac{2}{7} + \frac{3}{14}\right)$ | [2]<br>[2] |
| | c) $\frac{17}{38} \times \frac{57}{26} \div \frac{68}{39}$ | d) $\frac{\frac{35}{12} \times \frac{6}{7}}{\frac{1}{9} + \frac{7}{18}}$ | [2]<br>[3] |

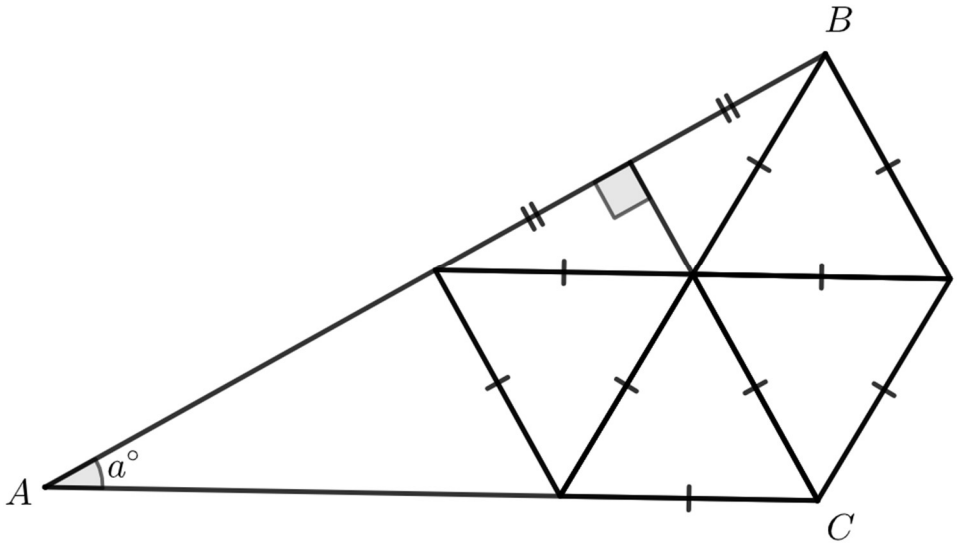
| | | | |
|---|---|---|---|
| **5.** | a) Solve $x - 5 = 18$. | b) Solve $6x + 9 = 10x - 19$. | [1]<br>[2] |
| | c) Solve $\frac{10}{1 - 3x} = -5$. | d) Solve $\frac{32}{3 + \sqrt{2x - 1}} = 4$. | [2]<br>[2] |

**6.**

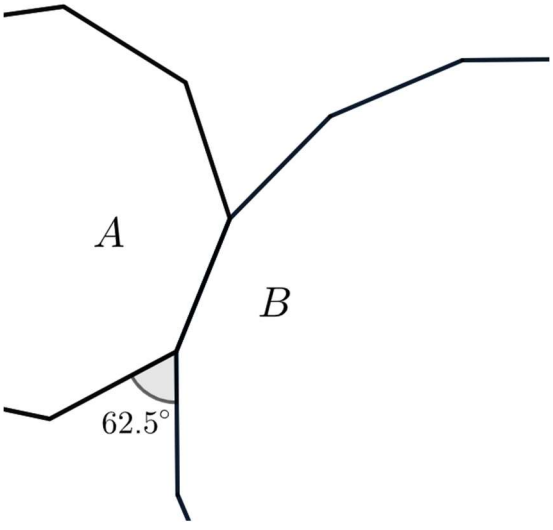
a) Find angle $a$. [2]




b) The diagram shows seven triangles, four of which are equilateral. $AB$ and $AC$ are straight lines. Find angle $a$. [2]




c) The diagram below partially shows two regular polygons, $A$ and $B$. $A$ has 9 sides. How many sides has $B$? [3]



**7.** $a = 4, b = 17, c = -9$ and $d = 3$.
Find the value of:

a) $\dfrac{3a - c}{d}$ [1]

b) $(b + c)^2 - a$ [1]

c) $ac + bd$ [1]

d) $(a + d)(a - d)$ [2]

e) $\dfrac{c^2 - b}{a^3}$ [2]

**8.**

a) 150 can be written as a product of its prime factors as: $150 = 2 \times 3 \times 5^2$. Write 180 as a product of its prime factors. [2]

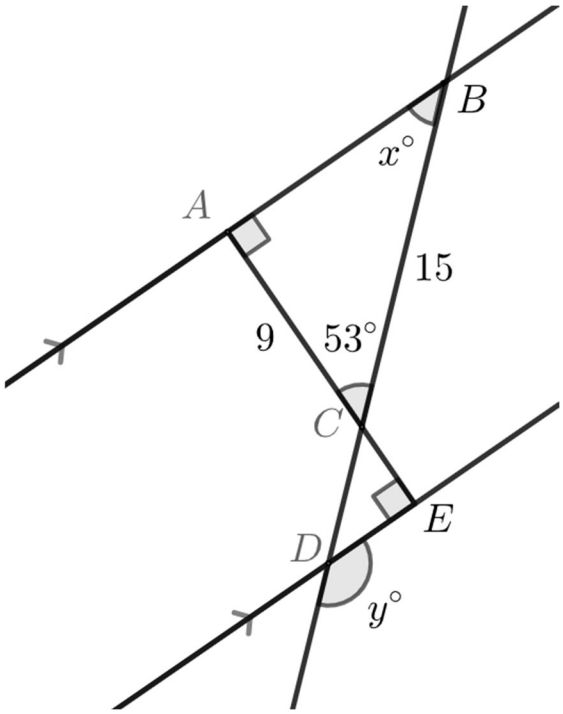
b) Write the highest common factor of 150 and 180 as a product of its prime factors. [2]

c) What is the ratio of 150 to the highest common factor of 150 and 180? [2]

d) Simplify the ratio $12 : 27 : 48$. [3]

e) Three friends win a quiz. Ella correctly answers twice as many questions as Fergus, who in turn correctly answered five times as many as Gary. They split the prize of £40 in proportion to how many questions they got correct. How much does Fergus receive? [3]

**9.**




In the diagram above $A$, $B$, $C$, $D$ and $E$ are points where lines intersect. The angle $ACB$ is 53°, rounded to the nearest whole number. $AC$ has length 9 and $BC$ has length 15. The line through $A$ and $B$ is parallel to the line through $D$ and $E$.

a) Find $x$ and $y$. [2]

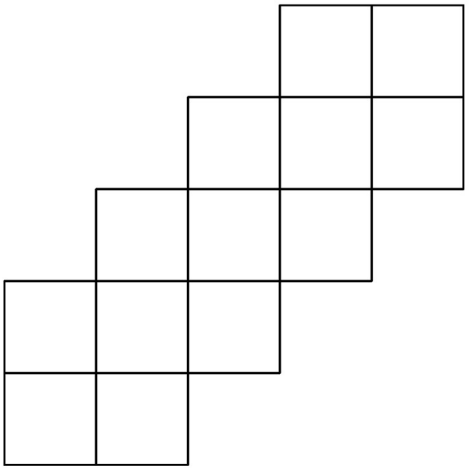
b) Find the length $AB$. [2]

The area of triangle $ABC$ is nine times larger than the area of triangle $CDE$.

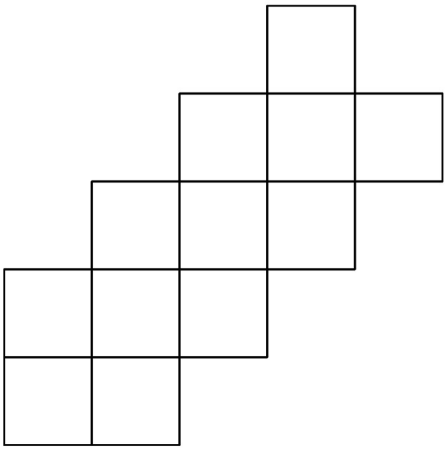
c) Find the length $CD$. [2]

| | | |
|---|---|---|
| **10**. | A list of five numbers is<br><br>0, 2, 5, 7, 11.<br><br>Two more whole numbers, $a$ and $b$, are added to this list so that both the mean and the median increase by two.<br><br>List every possible combination of $a$ and $b$. | [4] |

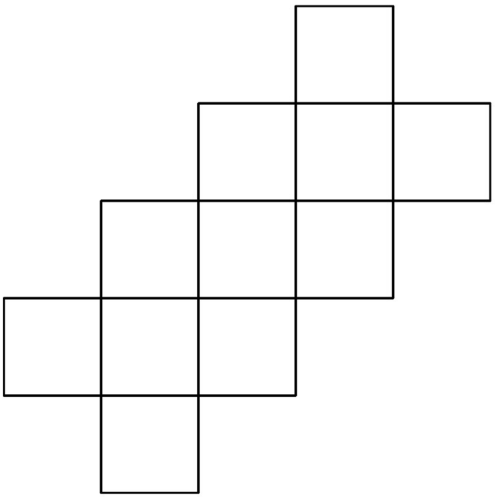
**11.** a) Exactly three of the thirteen squares in the shape below are to be coloured in. The resulting pattern must have rotational symmetry of order 2. In how many ways can this be done? [2]



b) Exactly three of the twelve squares in the shape below are to be coloured in. The resulting pattern must have a line of symmetry. In how many ways can this be done? *(You should give some indication as to how you arrive at your answer.)* [3]



c) Exactly three of the eleven squares in the shape are to be coloured in. The resulting pattern must have a line a symmetry. In how many ways can this be done? *(You should give some indication as to how you arrive at your answer.)* [3]



| | | |
|---|---|---|
| **12.** | Taylor is at a bench and Sundip is at a tree that are 8 metres apart. Taylor runs towards the tree at 6 m/s at the same time that Sundip walks towards the bench at 2 m/s.<br><br>a) How far are they from the bench when they meet? | [1] |
| | When Taylor reaches the tree she turns around and immediately runs back to the bench. When she reaches the bench she then runs back to the tree and then finally back to the bench, all the while maintaining a speed of 6 m/s.<br><br>Likewise, when Sundip reaches the bench, he turns around immediately and walks back to the tree, then back to the bench and then back to the tree, all the while maintaining a speed of 2 m/s.<br><br>b) How many times do Taylor and Sundip meet before Taylor finishes her journey? | [4] |
| | c) How does your answer to part b) change if the bench and tree are instead 800 metres apart? | [1] |

**13.** A sequence is defined by a starting whole number and the following rules:

- If the number is even it is divided by 2 to give the next number in the sequence.
- If the number is odd it is added to 1 to give the next number in the sequence.
- The sequence stops when it gets to 1.

For example, if I start at 20, I get the 8-term sequence:

$$20, 10, 5, 6, 3, 4, 2, 1.$$

a) How many terms are in the sequence that starts at 21? [1]

b) I start with a number bigger than 1000 and create a sequence with exactly 11 terms. What was my starting number? [2]

c) Find a number less than 100 that results in a sequence of length 14. [2]

d) The first term and the third term differ by 31. What could the first term be? [2]

**14.** To find the number of positive factors a number has you can look at the prime factorisation of the number, raise all the powers by one, and multiply the results together.

For example, to find the number of factors of $200 = 2^3 \times 5^2$ we calculate

$$(3 + 1) \times (2 + 1) = 12.$$

a) Find the number of factors of 450. [2]

A machine has a number on a display and four buttons which change the number on the display as follows:

A: increases the number by 50%
B: decreases the number by 10%
C: increases the number by 75%
D: increases the number by 96%

b) The starting number is 1000 and B is pressed three times. How many factors does the final number have? [3]

| | | |
|---|---|---|
| | c) The starting number is 250. A and D are pressed once each. How many factors does the final number have? | [3] |
| | d) The starting number is 10000. Is it possible to press buttons in such a way that the final number is a whole number with more than 80 factors? (*If yes, explain how; if no, explain why not.)* | [3] |