Name ……..…..……

School ……..…..……

WINCHESTER
COLLEGE

Winchester Election

Mathematics II

Wednesday 3rd May 2023

Time allowed: 1 hour 30 minutes

Total marks: 100

CALCULATORS ARE NOT ALLOWED.

Write your answers in this booklet. If you need additional space, please write on sheets of A4 paper and attach them to this booklet. You should show all your working so that credit may be given for partly correct answers.

Diagrams are not drawn to scale.

Do not be discouraged if you do not finish.

1. Evaluate:

a) $\dfrac{0.2}{0.05}$ [1]

b) $2\sqrt{0.25}$ [1]

c) 0.00002×50000 [1]

d) $116 + 9 \times 116 + 116$ [1]

e) $\dfrac{\sqrt{43} \times \sqrt{43}}{\left(\sqrt{129}\right)^2}$ [2]

f) Find 15% of 20% of 25% of 6000. [2]

g) Put the following numbers in ascending order:

$2^{14} \times 7^2$, $2^{13} \times 7^3$, $2^{11} \times 7^5$, $2^{12} \times 7^4$. [2]

2. Solve:

a) $\frac{36}{2x+1} = 4$ [2]

b) $7(2x + 1) - 3(6 - 3x) = -57$ [2]

c) $\sqrt[3]{\frac{9^2}{x}} + 3 = 6$ [2]

d) $\frac{3}{2x + 7} = \frac{5}{x + 21}$ [2]

e) $\frac{2x - \frac{2}{3}}{\frac{4}{5} - x} = 0$ [2]

f) Expand and simplify:
$(a^2 + b)(a^2 - b)$. [2]

g) Expand and simplify:
$3x(x + 2) - 2(x - x^2)$. [2]

h) Make x the subject of the formula:
$y = \left(\frac{x}{4}\right)^5 + 6$.
(Currently y is the subject.) [2]

i) Solve $\sqrt[3]{3\left(\sqrt{\frac{\sqrt{3x+3}}{\left(\frac{1}{3}\right)}}\right)} = 3$. [3]

3. A formula for p is

$$p = n^2 - n + 11.$$

a) Complete this table by finding the value of p when $n = 1$, $n = 3$, $n = 6$, $n = 9$ and $n = 11$. [2]

n	1	2	3	4	5	6	7	8	9	10	11
p		13		23	31		53	67		101	

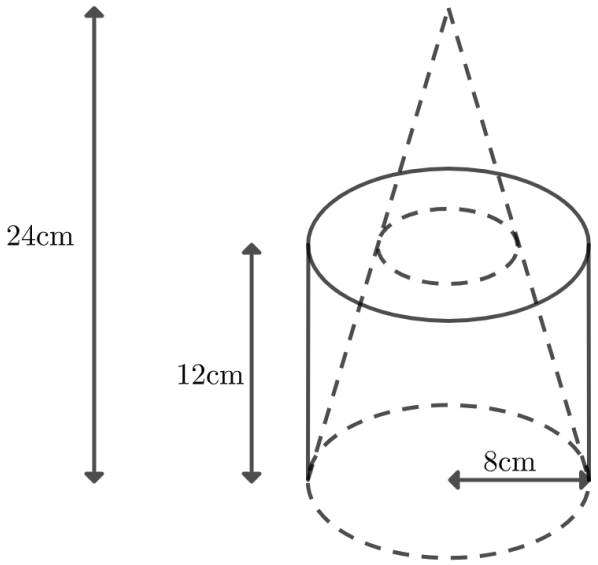
b) Circle the values of p in the table that are prime. [1]

c) Holly thinks that when n is any positive whole number the formula $q = n^2 - n + 41$ always gives values of q that are prime.

(i) Explain, using an example, why Holly is not correct. [2]

(ii) Now find one other example to show that Holly is not correct. [1]

4.	a) Consider the pair of numbers 30 and x. The highest common factor of the pair is 5 and the lowest common multiple of the pair is 210. What is x?	[2]
	b) The five numbers: *x, x, x, y, x+y,* have a median of 8 and a mean of 6. What is the range of the five numbers?	[3]

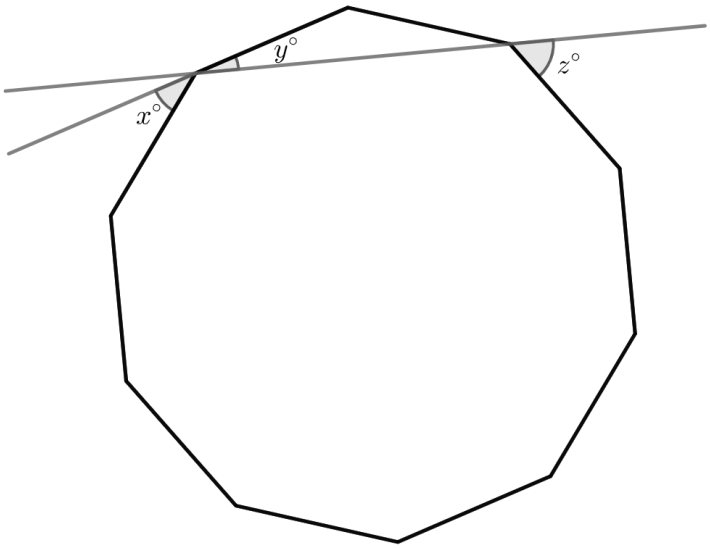
5.	A cone of height 24cm and base radius 8cm is shown, using dotted lines, in the diagram below. The top half of the cone is cut off and removed leaving a frustrum of height 12cm. The frustrum is placed in a cylinder, shown in the diagram with solid lines, also of height 12cm and base radius 8cm. Determine what fraction of the cylinder is not occupied by the frustrum. (A cone with height h and base radius r has volume $\frac{1}{3}\pi r^2 h$. A cylinder with height H and base radius R has volume $\pi R^2 H$.)  	[6]

6. An empty cuboid box has dimensions 3cm, 5cm and 6cm.

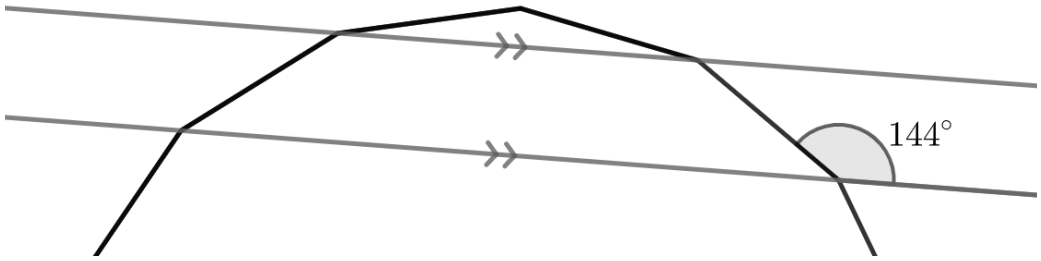
a) A fly, which can move freely in three dimensions, flies from one vertex of the box to the opposite corner. What is the shortest distance the fly could travel? Give your answer as an exact square root. [2]

b) A spider is on the outside of the box, and must stay on the surface of the box. The spider walks from one vertex of the box to the opposite corner. What is the shortest distance the spider could travel? [4]

7. a) The diagram below shows a line passing through the vertices of a regular decagon. Find x, y and z. (A decagon is a 10-sided shape.) [4]



b) The diagram below shows part of a regular polygon with n sides. Two parallel lines pass through vertices of the polygon as shown. Find n. [4]

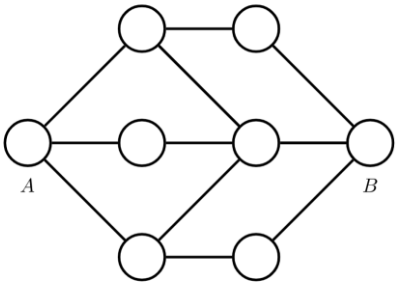


8. a) How many palindromic numbers are there between 1 and 1000, inclusive? (Palindromic numbers are those numbers that are the same if the digits are in reverse order. E.g. 3, 33 and 303 are all palindromic numbers.) [2]

b) The number of palindromic numbers between 1 and n, inclusive, is 198. What are the smallest and largest possible values that n can take? [3]

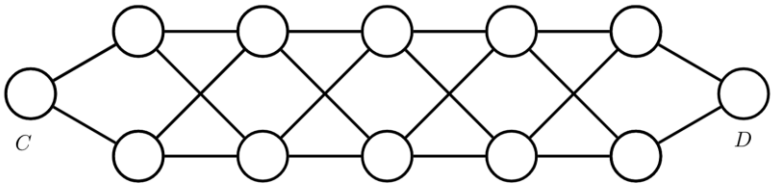
c) The number of palindromic numbers between 1 and p, inclusive, is q. The number of palindromic numbers between 1 and q, inclusive, is 18. What are the smallest and largest possible values that p can take? [4]

9. a) A beetle starts on circle A. Each second it crawls along a line to the circle at the other end of that line. It arrives at circle B after three seconds.



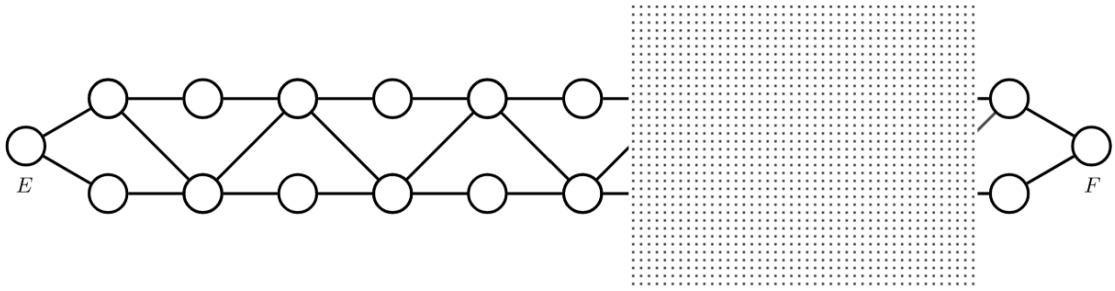
How many different paths could the beetle have followed? [2]

b) A second beetle starts at circle C, crawls along a straight line to a new circle every second and arrives at circle D after six seconds.



How many different paths could the beetle have followed? [3]

c) The pattern in the diagram continues in the region indicated by the dots (which is not to scale). The complete diagram contains $2n$ circles, including circles E and F.

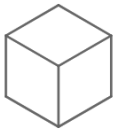


A third beetle starts at circle E, crawls along a line to a new circle each second and arrives at circle F after n seconds.

Given that there are more than 1000 paths the beetle could have followed, find the smallest possible value of n and the number of possible paths for that value of n. [4]

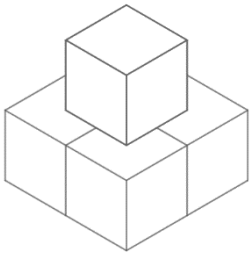
10. Pyramids are made out of 1 cm^3 blocks.

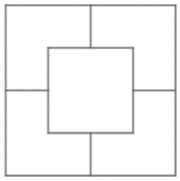
Pyramid #1 has one layer and uses one block.



Aerial view

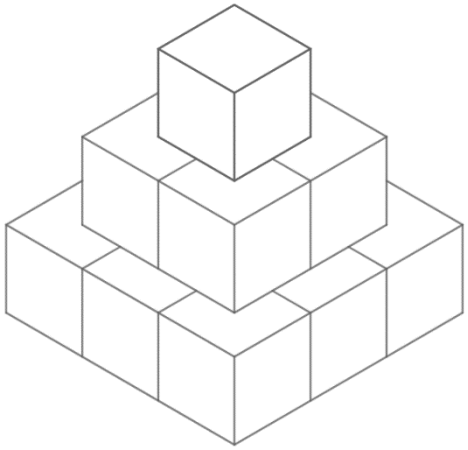
Pyramid #2 has two layers and uses five blocks.

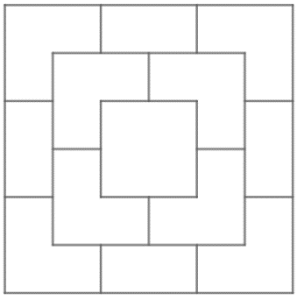




Aerial view

Pyramid #3 has three layers and uses fourteen blocks.





Aerial view

a) Pyramid #4 has four layers. How many blocks does it use? [1]

b) What is the total surface area (including the base) of Pyramid #3? [2]

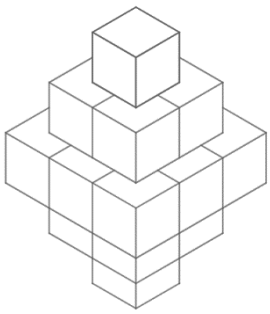
The sum of the first 100 positive whole numbers can be found by pairing them up in the following way.

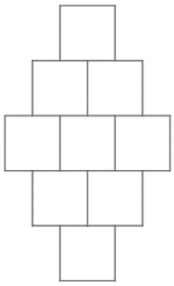
$$\begin{aligned} 1 + 2 + \cdots + 99 + 100 &= (1 + 100) + (2 + 99) + \cdots + (50 + 51) \\ &= 101 + 101 + \cdots + 101 \\ &= 101 \times 50 \\ &= 5050 \end{aligned}$$

c) Find the sum of the first 20 whole numbers. [2]

d) What is the total surface area (including the base) of Pyramid #20, which has twenty layers? [3]

A new shape is also made out of 1cm^3 blocks. Shape #3 is made by two Pyramid #3s now sharing the same third layer (the base of the pyramid), with one pyramid pointing upwards and one pointing downwards.

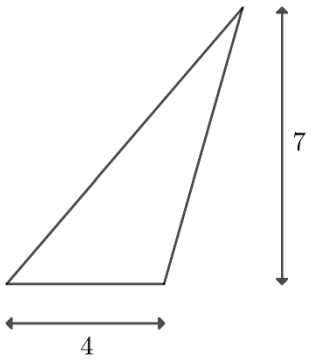




Side view

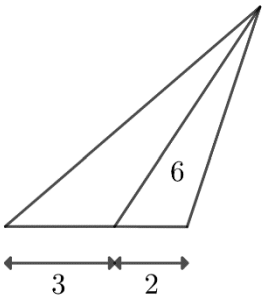
e) What is the total surface area of Shape #4, made by two Pyramid #4s sharing the same fourth layer, with one pyramid pointing upwards and one pointing downwards? [3]

11. a) Find the area of the triangle shown on the right. [1]

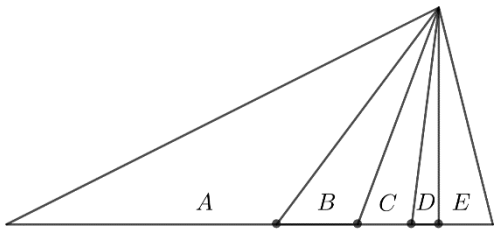


b) In the diagram, 6 represents the area of the right hand part of the triangle.

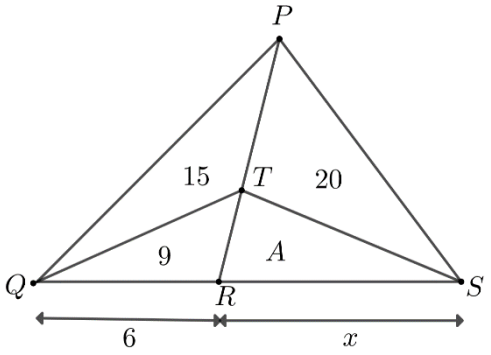
Find the total area of the large triangle. [2]



c) In the diagram, the base of the triangle has been divided in the ratio $6 : 3 : 2 : 1 : 2$ by points which are joined to the top by lines.
Which two of the regions A, B, C, D and E have a combined area equal to half the area of the whole triangle? [1]



d) In the diagram, 15, 20, 9 and A represent the areas of the four parts of triangle PQS. PTR is a straight line. Write the ratios below in the form $a : b$, where a and b are whole numbers and $a : b$ is fully simplified.



(i) $15 : 20$ [1]

(ii) $9 : A$ [1]

(iii) $6 : x$ [1]

e) The diagram shows a triangle ADF with $AB = BC = CD$ and $BG = GF$. AGE is a straight line and the lengths of DE and EF are x and 12 respectively. Find x, giving a brief explanation of your reasoning. [4]